

Propaganda and Repression in Diverse Societies*

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Repression and information manipulation are two primary tools employed by authoritarian regimes. We draw on recent advances in the economic theory of information to examine how repression and propaganda complement each other: when the regime's opponents face harsher repression, persuasion is more effective. When the regime can target less supportive citizens for repression, it can manipulate the rest of the population more intensively. Moreover, propaganda is less effective in diverse societies; consequently, the regime has to rely more on repression and can benefit from reducing diversity. In the era of information autocrats, our model highlights the critical role of repression.

Keywords: Authoritarian regimes, repression, propaganda, diverse societies, Bayesian persuasion.

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Introduction

After the collapse of the Soviet Union and other socialist dictatorships in the late 1980s–early 1990s, authoritarianism as a form of government appeared to be on its way out (Fukuyama, 1992). Three decades later, authoritarian regimes have proved remarkably durable, adapting to the challenges of globalization, technological progress, and increased awareness of civil rights (Applebaum, 2024). The critical element of this adaptation was the novel use of two tools of authoritarian control, information manipulation and repression.

Repression and propaganda have always played an important role in keeping autocrats in power (Machiavelli, 1532; Wintrobe, 1990; Svoboda, 2012). In the 20th century, information manipulation was a cornerstone of totalitarian rule in Hitler's Germany, Stalin's Russia, and Mao's China (Arendt, 1951; Friedrich and Brzezinski, 1956; Dimitrov, 2023; Harrison, 2023). With the demise of totalitarian dictatorships, propaganda is used not for ideological indoctrination, but to maintain the leader's reputation as a strong and competent hand (Treisman, 2011; Lorentzen, 2013; Huang, 2018; Rozenas and Stukal, 2019; Guriev and Treisman, 2019; Gratton and Lee, 2024). In the 21st century, leaders manipulate information by controlling the state media (Rozenas and Stukal, 2019), co-opting or pressuring independent media outlets (McMillan and Zoido, 2004; Szeidl and Szucs, 2021), and censoring unfavorable news (Lorentzen, 2014; Shadmehr, 2014; Gehlbach et al., 2024).¹ Instead of arresting millions as Stalin, Hitler, or Mao, or carrying out public executions as Pol Pot, Hussein, or Nguema, they primarily rely on selective censorship, digital surveillance, and sophisticated propaganda. They are, as Guriev and Treisman (2019, 2020, 2022) put it, *informational autocrats*.

Yet *repression* remains a critical instrument in the autocrat's arsenal (Tyson, 2018; Montagnes and Wolton, 2019; Rozenas, 2020). Following the 2020 protests, Belarus' Alexander Lukashenko had more than 30,000 people arrested and hundreds given long jail terms, a more than ten-fold increase over the average number of political prisoners during the previous decade (Way and Tolvin, 2023; Anisin, 2024). After the start of Russia's full-scale invasion of Ukraine in February 2022, Putin's regime has doubled down on repression: tens of thousands of people opposing the war and Putin's rule were arrested or forced out of the country; hundreds if not thousands received sentences in the range of 8–25 years, numbers unheard of since Stalin's

¹Attempts to control media and manipulate information extend beyond autocracies. Examples in democracies abound, from Argentina (Di Tella and Franceschelli, 2011) to Mexico (Stanig, 2015) to Italy (Durante and Knight, 2012) to the United States (Qian and Yanagizawa-Drott, 2017; Gentzkow and Shapiro, 2008).

years (Treisman, 2022; Stoner, 2023). At the same time, information manipulation that has long been Putin's weapon of choice against the domestic opposition (Gehlbach, 2010; Treisman, 2011; Rozenas and Stukal, 2019; Guriev and Treisman, 2022) increased dramatically after the invasion.

In this paper, we study the interrelationship between the two primary tools of authoritarian control using recent advances in the economic theory of persuasion (e.g., Kamenica and Gentzkow, 2011; Gentzkow and Kamenica, 2014; Alonso and Câmara, 2016a,b; Galperti, 2019; Kolotilin et al., 2017; Kolotilin, Mylovanov and Zapechelnyuk, 2022; Lipnowski, Ravid and Shishkin, 2022). We do not claim a contribution to economic theory; instead, we apply the existing theoretical results to understand how modern dictators combine information manipulation and repression to stay in power, and then check how robust our implications are.

There are two main findings: First, when citizens are more heavily repressed or expect a harsher punishment for opposing the regime, the autocrat engages in more information manipulation. In other words, repression and propaganda are natural complements.² This result is robust: it does not matter whether citizens expect repression if they actively oppose the regime, or the regime has the ability to sort out those who are skeptical and purge them. In either case, information manipulation complements repression. Second, when society is more diverse, the effect of information manipulation is smaller.

The basic logic behind the complementarity result is as follows. At a given propaganda level, there is a marginal citizen who is indifferent between protesting against the regime and not. Those citizens who consider the leader less competent than this marginal citizen do protest against the regime; those who have more faith in the leader stay home. If the leader increases the level of propaganda, this citizen will protest because she is already manipulated to the maximum extent possible. Now, suppose that the repression increases. The previously marginal citizen is now infra-marginal and is unwilling to protest as she fears the increased repression. As a result, the leader gets extra leeway when it comes to propaganda, allowing the leader to persuade some of those who were not persuadable at the previous level of propaganda.

Our second result establishes that when citizens' beliefs are more diverse, the autocrat finds it optimal to decrease the propaganda level. When citizens have identical beliefs about the regime, the optimal strategy of information manipulation involves intensifying the level of

²We use the terms *propaganda* and *information manipulation* interchangeably. In other contexts, *propaganda* might be used more restrictively, referring to a particular type of information manipulation, or vice versa, more loosely, covering techniques that are not modeled as information manipulation by economic theorists.

propaganda just enough to make citizens indifferent between protesting and not. However, under such a strategy, a slight increase in citizens' belief diversity results in the autocrat losing the support of half the citizens. To gain broader support, the autocrat must appeal to more skeptical citizens, which requires adopting less intense, but more credible, propaganda.

The implication that a more diverse society is more resistant to information manipulation has an underlying logic similar to the one that connects information manipulation and repression. Under the propaganda level tailored for the whole population, the skeptics would have been "underpersuaded" from the perspective of the leader. Therefore, the leader increases the intensity of propaganda if the most disloyal elements of the society are removed. Thus, the dictator does not face the choice of repression versus propaganda but rather benefits from those reinforcing each other. Our contribution here is to illustrate how this reasoning generalizes under a novel partial order on distributions, which captures the idea of diversity of beliefs.

Repressions have been shown to change citizens' behavior.³ [Montagnes and Wolton \(2019\)](#) and [Rozenas \(2020\)](#) use communist purges in Stalin's Russia and Mao's China to demonstrate this effect. Physical elimination as in [Esteban, Morelli and Rohner \(2015\)](#) changes the composition of the society; other forms of political disenfranchisement might also have the same effect. In addition to mass executions, Stalin relocated hundreds of thousands from places where they were a political threat to distant regions of Russia. In most cases, Stalin's mass repression campaigns were organized around broad ethnic or social categories ([Gregory, Schröder and Sonin, 2011](#)); in our model, this would correspond to the leader repressing citizens based on imperfect information about their initial beliefs. In the realm of democratic politics, [Glaeser and Shleifer \(2005\)](#) shows that the incumbent politician could deliberately choose policies that drive voters who oppose him out of the district. Our theory also applies to such situations. After repression, the rest of society will be exposed to more information manipulation.

To model information manipulation, we use the basic model of Bayesian persuasion ([Kamenica and Gentzkow, 2011](#); [Bergemann and Morris, 2019](#); [Little, 2023](#)) on an audience with heterogeneous priors ([Alonso and Câmara, 2016b](#); [Laclau and Renou, 2017](#); [Galperti, 2019](#); [Kartik, Lee and Suen, 2021](#); [Onuchic and Ray, 2023](#)).⁴ Compared to other communication

³[Arendt \(1951\)](#) makes a distinction between *dictatorial* terror, aimed against well-identified opponents of the regime, from an all-pervasive *totalitarian* terror of purges, mass executions, and concentration camps. Modern theories of repression with strategic targeting and selection include [Myerson \(2015\)](#), [Tyson \(2018\)](#), and [Dragu and Przeworski \(2019\)](#).

⁴Also see [Kosterina \(2022\)](#), who studies Bayesian persuasion with unknown priors and a worst-case-maximizing sender, and [Shimoji \(2022\)](#), who develops a linear programming approach to Bayesian persuasion with heterogeneous priors.

protocols, the model of Bayesian persuasion assumes fuller commitment on behalf of the sender.⁵ This makes perfect sense in our application: dictators do not edit news in real time. Instead, they pass restrictive laws, establish institutions of censorship, and appoint loyal editors to control the flow of information. The choice of an institutional bias or an editor of known ideological preferences corresponds to the choice of the main control parameter in the model. Still, the assumption of full commitment is not necessary: in Section 5, we use the fundamental results of [Lipnowski, Ravid and Shishkin \(2022\)](#) to show that our substantive implications hold if there is a chance that the leader can renege on his commitments.

In addition to substantive reasons, there are theoretical advantages in using the Bayesian persuasion model instead of other models of information manipulation.⁶ Most importantly in our case, the model allows one to study the *maximum propaganda*: it provides the upper limit on the amount of persuasion that can be done via any information exchange between a sender and a receiver. At the same time, our qualitative results easily translate to other information exchange models such as cheap talk in [Crawford and Sobel \(1982\)](#), verifiable messaging in [Milgrom \(1981\)](#) (see also [Titova and Zhang, 2025](#)), and signaling in [Spence \(1973\)](#). Although the machinery of a propaganda and repression model based on other communication protocols would be different, the qualitative intuition is the same. In Section 5, we discuss why our basic intuition carries over to other communication protocols and the role of other assumptions. In particular, we show that similar technical results can be obtained in the framework of the concurrent theoretical paper [Kolotilin, Mylovanov and Zapechelnyuk \(2022\)](#).

An important assumption of our model is that the leader organizes information manipulation as public communication: he establishes an institution which learns the true state of the world and makes a public report. To establish the robustness of our substantive findings, we explore whether the leader could do better if it were possible to target different citizens with different messages. Of course, if the leader can compel each individual to consume the news specifically tailored to this individual, the effect of persuasion will be greater than in the case of public communication. In fact, this would be the best that the leader could do. However, if

⁵In [Guriev and Treisman, 2020](#), the leader knows her competence, which means that she has zero commitment power. (Though [Guriev and Treisman, 2020](#)'s theoretical model does not exclude complementarity, it treats repression and information manipulation primarily as substitutes.)

⁶Bayesian persuasion is a rapidly expanding field in economic theory. [Wang \(2015\)](#) and [Chan et al. \(2019\)](#) also compare public and private persuasion, where the sender knows the heterogeneous preferences of the receivers. [Alonso and Câmara \(2016a\)](#) and [Inostroza and Pavan \(2022\)](#) study public persuasion toward heterogeneous receivers with known preferences, while [Bardhi and Guo \(2018\)](#), [Arieli and Babichenko \(2019\)](#), [Taneva \(2019\)](#) and [Mathevet, Perego and Taneva \(2020\)](#) study private persuasion toward heterogeneous receivers with known preferences. On private persuasion towards heterogeneous receivers with private preferences, [Guo and Shmaya \(2019\)](#) characterize the optimal information structure, and [Heese and Lauer mann \(2024\)](#) study a voting setting.

the leader has to worry about citizens' access to other sources of information, for example, to messages tailored to other citizens, then the leader's power is limited. In Section 5.5, we use the celebrated result of Kolotilin et al. (2017) to demonstrate that, if the leader has to make the consumption of news incentive compatible, the possibility of private persuasion does not add to the leader's persuasion power. With this result in hand, Proposition 7 justifies our assumption that the leader adheres to the public persuasion mechanism. Substantively, the result explains why authoritarian regimes use blank, one-size-fits-all messaging in situations where they cannot make sure that there is no information exchange between different groups of individuals.

The rest of the paper is organized as follows. Section 2 sets up our model. Section 3 studies the main case, when the leader optimally combines repression and persuasion. Section 4 deals with the case where the leader can repress the opposition ex ante. Finally, Section 5 discusses the robustness of our results to alternative technical assumptions.

2 Setup

We consider a government that tries to dissuade citizens from protesting against it using information manipulation and repression. Citizens are heterogeneous with respect to their attitudes towards the government.

2.1 The Leader and Citizens

There is a sender s (the *leader*) and a continuum of receivers $I = [0, 1]$ (*citizens*, with a generic citizen denoted by i). The citizens' decision whether to protest against the leader depends on an unknown characteristic of the leader, which we refer to as *competence* for concreteness. Formally, we assume that there is an unknown state of the world denoted by $\omega \in \{incompetent, competent\}$. Citizen i 's prior on the leader's competence is

$$\mu_i = \Pr_i(\omega = competent),$$

whereas the leader's prior is $\mu_s \in (0, 1)$.

Citizens have *heterogeneous priors* about the leader's competence. The density function for the distribution of priors among citizens is denoted by $f(\mu)$, with $F(\mu)$ denoting the corresponding cdf. The heterogeneity in priors reflects both citizens' varying degrees of knowledge about

the leader's quality and their genuine disagreement about what it means to be a competent leader. We assume that both the leader's prior, μ_s , and the distribution of priors among citizens, f , are common knowledge.

Throughout the paper, we maintain the following assumption on the distribution of priors:

Assumption 1. Probability density function f has full support on $[0, 1]$, is continuously differentiable with $f(0) = f(1) = 0$, and is strictly log-concave, that is, $\frac{\partial^2}{\partial \mu^2} \log f(\mu) < 0$ for all $\mu \in [0, 1]$.

The assumption that the density function f is log-concave is a mild one. For example, the following distributions satisfy this assumption: uniform, (truncated) normal, and beta (with both parameters ≥ 1), among others—see [Bagnoli and Bergstrom \(2005\)](#) for a list.

Given her information on the state of the world, ω , each citizen $i \in I$ decides whether to protest against the leader or stay home; we let $a_i \in \{protest, stay\}$ denote this action. Citizen i 's payoff is denoted by $u(a_i, \omega)$ and is given in [Table 1](#).

	$\omega = incompetent$	$\omega = competent$
$a_i = protest$	$1 - r$	$-r$
$a_i = stay$	0	0

Table 1. Citizen payoffs given repression level r .

The payoff from staying home is normalized to zero. Citizens who protest face a cost of r regardless of the leader's competence and get a benefit (normalized to one) only if the leader is incompetent. Here, $r \in (0, 1)$ parameterizes the punishment for participating in protest, or, equivalently, the *repression* level. Citizens prefer to stay home if they think the probability that the leader is competent is sufficiently high. (In that sense, their prior probability captures not only the extent of their individual knowledge about the leader but also their attitude toward him.)

The leader incurs a cost for every citizen who protests. Specifically, the leader's payoff from protest is given by

$$u_s(\{a_i\}_{i \in I}) = - \int_{i \in I} \mathbb{1}_{\{a_i = protest\}} di. \tag{1}$$

2.2 Information Manipulation

To persuade citizens to stay home, the leader uses a public persuasion mechanism. That is, the leader commits to an information structure $\{\sigma(\cdot|\omega)\}_{\omega \in \{0,1\}}$, where

$$\sigma(\cdot|\omega) \in \Delta(M) \quad \text{for all } \omega \in \{0,1\},$$

and the realized message, $m \in M$, is publicly observable to each citizen and the leader. We assume that $|M|$ is large enough that there are sufficiently many action recommendations for each receiver. As we show later, under Assumption 1, the leader uses at most two messages.

2.3 Repression

We model the leader's choice of repression as follows. Before committing to the propaganda level, the leader can choose the repression level $r \in (0, 1)$ at a cost. Although repression increases the expected cost of opposing the leader, it is costly for the ruler. We assume that if the leader chooses the level of repression $r \in [0, 1]$, he incurs a $\phi \cdot c(r)$ cost, where $\phi > 0$ parameterizes the cost of repression and $c(r)$ increases in r . We impose the following assumption on the cost of repression, which ensures that the solution is well-behaved and interior:

Assumption 2. $c : [0, 1] \rightarrow \mathbb{R}$ is strictly increasing and strictly convex, with $c(0) = c'(0) = 0$ and $\lim_{r \rightarrow 1} c'(r) = \infty$.

In addition, the leader knows that the choice of r will be accompanied by the optimal propaganda level $\beta^*(r)$ and the payoff $V^*(r)$ from information manipulation. Thus, the leader's choice of repression, r^* , is the solution to the following problem:

$$r^* \in \arg \max_{r \in [0,1]} \{V^*(r) - \phi \cdot c(r)\}. \quad (2)$$

2.4 Diversity

The leader might benefit from reducing the diversity of opinions before engaging in propaganda. To study this, we start with our definition of what it means to be a less (or a more) diverse society, which is a condition on the distribution of priors f , and then discuss how less diversity is linked to more propaganda. Our measure is a novel partial order on probability distributions.

Definition 1. Consider two log-concave distributions with densities f_1 and f_2 supported on a common compact set. f_2 is *less diverse* than f_1 if

$$f_2(x) = \alpha (f_1(x)) \quad \text{for all } x, \quad (3)$$

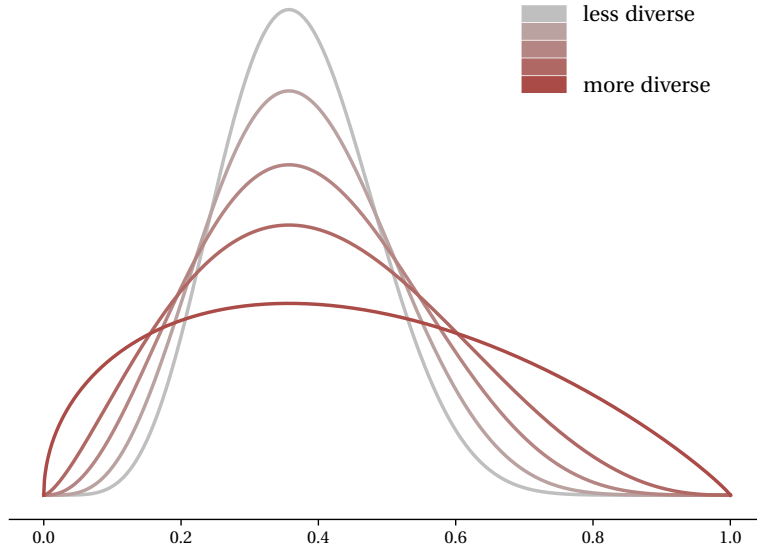


Figure 1. The diversity order.

for some strictly increasing and convex function $\alpha : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ with $\alpha(0) = 0$.

This partial order has an intuitive interpretation. Since α is convex and f_1 and f_2 both have to integrate to one, transforming f_1 by α magnifies the parts of f_1 with higher values and shrinks the parts with lower values. Moving from f_1 to f_2 thus moves the mass from parts of the distribution that initially have a smaller mass to parts with a larger initial mass. In other words, f_2 looks like f_1 , but with higher peaks and deeper troughs. But since f_1 is log-concave (that is, single-peaked), most of its mass is concentrated around its peak. Therefore, f_2 has even more mass in the center and even less mass in the periphery relative to f_1 ; that is, f_2 represents a less diverse society than f_1 . Figure 1 illustrates the probability density functions for a set of single-peaked Beta distributions that are ranked in diversity order.⁷

Members of many parametric families of distributions can be ordered by their diversity. Two examples follow:

Example 1. Consider two single-peaked Beta distributions

$$f_1 = \text{Beta}(\alpha_1, \beta_1),$$

$$f_2 = \text{Beta}(\alpha_2, \beta_2),$$

⁷Johnson and Myatt (2006)'s *rotation order* is a related partial order, which also ranks distributions in terms of their dispersion or heterogeneity. The main difference between the two orders is that Johnson and Myatt (2006) consider rotations of a cumulative distribution function around a given point, whereas in our partial order the rotation point itself depends on the distribution function. The endogeneity of the rotation point to the distribution function is crucial for our comparative statics results. It ensures that the rotation point is always in the appropriate range for an increase in diversity to have an unambiguous effect on the propaganda level.

where $\frac{\alpha_1-1}{\alpha_1+\beta_1-2} = \frac{\alpha_2-1}{\alpha_2+\beta_2-2}$. If $\alpha_1 \geq \alpha_2$, then f_1 is less diverse than f_2 , while if $\alpha_1 \leq \alpha_2$, then f_2 is less diverse than f_1 . In particular, any two single-peaked Beta distributions with the same mode are ranked according to the diversity partial order.

Example 2. Consider the following truncated normal distributions on $[0, 1]$:

$$\begin{aligned} f_1 &= \text{TruncatedNormal}(\mu, \sigma_1^2), \\ f_2 &= \text{TruncatedNormal}(\mu, \sigma_2^2). \end{aligned}$$

If $\sigma_1^2 \geq \sigma_2^2$, then f_2 is less diverse than f_1 .

3 Analysis

We start our analysis with a standard exercise to calculate the optimal degree of information manipulation that the leader chooses if propaganda is the only tool at his disposal. Then, we derive comparative statics with respect to the level of repression.

3.1 Optimal Persuasion

Given a repression level $r \in (0, 1)$, what is the leader's optimal information structure? Technically, we follow a standard approach to analyzing models of information manipulation (Kamenica and Gentzkow, 2011; Alonso and Câmara, 2016b). Once the sender has committed to an information design, his posterior belief conditional on signal realizations can be used as a parameter that controls receivers' beliefs. Specifically, we construct the **value function** of the leader as a function of his own posterior belief $\mu = \Pr_s(\omega = \text{competent} | m)$ and then use the concavification approach of Kamenica and Gentzkow (2011).⁸

The leader's payoff can be written as a function of his own posterior. Suppose that the leader's posterior conditional on observing the message is $\mu \in [0, 1]$. Because the leader and citizens observe (and update their beliefs based on) the same public message m , as long as we know the priors of the leader and citizen i , we can back out the posterior of citizen i from the leader's posterior. Using the Bayes formula, citizen i has the following posterior:⁹

$$\mu'_i = \frac{\mu \frac{\mu_i}{\mu_s}}{\mu \frac{\mu_i}{\mu_s} + (1 - \mu) \frac{1 - \mu_i}{1 - \mu_s}}. \quad (4)$$

⁸Note that leader perfectly learns the realization of the state. μ is what the leader's posterior would be had he not learned the state and instead only observed the message sent by the mechanism. For us, μ simply serves as a theoretical device that helps characterize the optimal persuasion mechanism.

⁹This expression is derived in Alonso and Câmara (2016b).

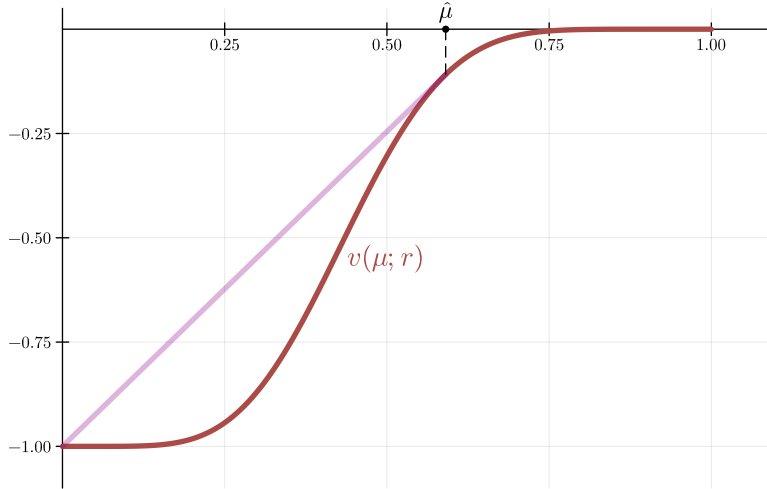


Figure 2. The Leader's Value Function and its Concavification.

Consider citizen i with posterior μ'_i . Given the payoff in Table 1, she protests if and only if

$$(1 - \mu'_i) \cdot (1 - r) + \mu'_i \cdot (-r) \geq 0,$$

which simplifies to $\mu'_i \leq 1 - r$.¹⁰ Substituting (4) into this inequality and rearranging terms, we can conclude that any citizen i with prior

$$\mu_i \leq \frac{1 - \mu}{1 - \mu + \mu \frac{r}{1-r} \frac{1-\mu_s}{\mu_s}}$$

protests. Given (1), the leader's value function as a function of his own posterior, μ , is

$$v(\mu; r) \equiv -F\left(\frac{1 - \mu}{1 - \mu + \mu \frac{r}{1-r} \frac{1-\mu_s}{\mu_s}}\right). \quad (5)$$

The optimal solution relies on the characterization of the concave closure of $v(\mu; r)$. An inspection of (5) immediately reveals that $v(\mu; r)$ is strictly increasing in μ , with $v(0; r) = -1$ and $v(1; r) = 0$. The next result characterizes the shape of the value function.

Lemma 1. $v(\mu; r)$ is strictly S-shaped, that is, there is some $\tilde{\mu} \in [0, 1]$ such that $v(\mu; r)$ is strictly convex for $\mu \in [0, \tilde{\mu}]$ and strictly concave for $\mu \in [\tilde{\mu}, 1]$.

Figure 2 depicts the leader's payoff as a function of his posterior. Visual inspection reveals that, from the leader's standpoint, the optimal information structure invokes two posteriors.

¹⁰Our notation suggests that a citizen protests when indifferent. Since f is continuously differentiable by Assumption 1, it has no mass points, so the measure of citizens with posteriors exactly equal to $1 - r$ is zero. Therefore, this choice of notation is inconsequential for the analysis.

This can be achieved by sending two messages about the leader’s competence: $m \in \{bad, good\}$. Moreover, one of the posteriors in the support is $\mu = 0$, i.e., one of the messages perfectly reveals that the leader is incompetent. This can be achieved by setting $\sigma(m = good|\omega = competent) = 1$ in the optimal policy.¹¹ Therefore, the optimal policy is characterized by a single-dimensional object:

$$\beta = \sigma(m = good|\omega = incompetent) \in [0, 1].$$

The value of β captures the extent of information manipulation. It is the probability that citizens receive the message that “the leader is good” when the leader is, in fact, bad. Since the message $m = bad$ perfectly reveals $\omega = incompetent$, it stands as an admission that the leader is bad. When the leader is bad, admission occurs with probability $1 - \beta$. One standard interpretation is that β is a measure of the extent of media censorship by the leader. When β is high, citizens are unlikely to hear the truth about the leader. Instead, the probability of hearing a positive message about the leader is high.

The leader faces a trade-off when solving his information design problem. Let $\beta^*(r)$ denote the level of propaganda chosen by the leader under the optimal policy as a function of r . Then the leader’s (subjective) expected payoff under the optimal information manipulation policy is given by

$$V^*(r) \equiv (1 - \mu_s)(1 - \beta^*(r)) \cdot v(0; r) + (\mu_s + (1 - \mu_s)\beta^*(r)) \cdot v\left(\frac{\mu_s}{\mu_s + (1 - \mu_s)\beta^*(r)}; r\right). \quad (6)$$

The expression (6) exhibits the main trade-off in the leader’s problem. A higher level of $\beta^*(r)$ leads to a higher frequency of good news and therefore to more *intense* propaganda. This is observed through the first and second appearances of $\beta^*(r)$ in (6): A higher $\beta^*(r)$ increases the probability that the leader’s payoff is $v\left(\frac{\mu_s}{\mu_s + (1 - \mu_s)\beta^*(r)}; r\right)$, rather than $v(0; r)$. On the other hand, a higher frequency of good news means, by definition, that the good news is less surprising, and so, moves the citizens’ beliefs by less—this is seen through the last appearance of $\beta^*(r)$ in Equation (6). Therefore, more intense propaganda is inevitably less *credible*. The leader chooses $\beta^*(r)$ to optimally resolve the trade-off between intensity and credibility.

In the optimal strategy, the leader chooses a cut-off citizen with belief $\mu^* \leq \mu_s$ such that (i) citizens with priors $\mu_i \geq \mu^*$ act based on the message and (ii) citizens with priors $\mu_i < \mu^*$

¹¹When $\mu_s \geq \hat{\mu}$ in Figure 2, the optimal policy does not reveal any information. In this case, any policy with two messages where $\sigma(m = good|\omega = incompetent) = \sigma(m = good|\omega = competent)$ is optimal. Among the many optimal policies, we choose the one where $\sigma(m = good|\omega = incompetent) = 1$, so that the “bad” message is never sent and the beliefs following $m = bad$ are free. Consequently, even in this case we can choose the posterior following $m = bad$ to be zero.

protest regardless of the message. A higher $\beta^*(r)$ increases μ^* (that is, the fraction of citizens who always protest), which is bad for the leader, but it decreases the probability of protest for those with $\mu \geq \mu^*$ (who protest only when they hear bad news), which is good. Thus, $\beta^*(r)$ is the level of propaganda that resolves this trade-off.

3.2 Propaganda and Repression

Having characterized the optimal persuasion mechanism in our setting, we can now focus on our main question: How does repression complement information manipulation? Our first proposition is a straightforward result that deals with comparative statics with respect to r , the repression level. In spite of its technical triviality, this result is the core of our political-economic analysis.

Proposition 1. *The extent of information manipulation, $\beta^*(r)$, increases with the level of repression, r .*

Intuitively, repression helps the leader overcome its credibility problem. The leader's desire for more intense propaganda is restrained only by the need to appear at least somewhat credible. If propaganda is too intense, then a large fraction of citizens choose to protest regardless of the messages sent by the leader. A higher level of repression relaxes this constraint by coercing more citizens to stay home. This gives the leader the freedom to increase the propaganda intensity. In other words, repression can act as a substitute for credibility.¹²

Next, we analyze the optimal choice of repression for the leader. Repression is any act by the leader that increases the cost of opposing him. This includes harassment, intimidation, imprisonment, torture, or execution. The incidence of repression is determined by a citizen's choice of action and therefore can be thought of as the expected cost of opposing the leader. That is important as even in totalitarian dictatorships only a small share of the population is actually punished. The Great Terror, 1937–1939, the most intensive years of Stalin's repression, had about 650,000 people executed, and 800,000 more sent to jails and labor camps. These are staggering numbers and, yet, only about 1% of the USSR population at the time (Conquest, 2008).¹³

¹²In a recent working paper, Curello and Sinander (2024) derive a similar comparative static result with respect to the extent of information manipulation. Their comparative statics result is based on a partial order on value functions. In contrast, our comparative statics are based on a more primitive parameter of the model, r . Similarly, Proposition 2 of Kolotilin, Mylovanov and Zapechelnyuk (2022) presents a comparative static result based on the *shifts* of the value functions.

¹³In line with the logic of our model, Stalin's Great Terror was accompanied by a massive propaganda campaign (Conquest, 2008; Kotkin, 2017). In another example, propaganda has played a critical role ever since the Chinese Communist Party took over in 1949, yet it became

We are interested in how the leader's chosen repression and propaganda levels are affected by the cost of repression. Our next result shows that, naturally, when repression is cheaper, it is used more intensely. Indeed, by Assumption 2, the leader's objective function $V^*(r) - \phi \cdot c(r)$ is strictly submodular in ϕ and r . The standard supermodularity arguments (e.g., Theorem 5 of Milgrom and Shannon, 1994) show that the repression level, r^* , is decreasing in the cost of repression, ϕ .¹⁴ By Proposition 1, $\beta^*(r^*)$ increases in r^* , and therefore decreases in ϕ . The following proposition summarizes the above argument.

Proposition 2. *The propaganda level, $\beta^*(r^*)$, is decreasing with the cost of repression, ϕ .*

The intuition behind this result is that a leader can use repression as a substitute for credibility. Intense propaganda is ineffective if many citizens do not find the leader's messages credible and protest regardless of his statements. To avoid this, the leader can either lower the intensity of propaganda to build credibility or use repression to increase the cost of protest. A leader with access to more effective repression methods (that is, a smaller ϕ) is less concerned with maintaining credibility, since he can compensate for any loss of trust with heavy-handed repression. Proposition 2 shows that *when the cost of repression is lower a higher intensity of repression is accompanied by a higher level of propaganda*. Propaganda and repression are complements.

4 Repression and Propaganda in Diverse Societies

So far, we have assumed that repression affects everyone who protests against the leader. In this section, we relax this assumption. We show that a society with more diverse attitudes towards the dictator is harder to manipulate. Then, we show that the higher is the dictator ability to detect and eliminate the opponents of the regime, the more manipulable the rest of citizens are. In other words, the complementarity between information manipulation and repression holds not only when citizens are punished for opposing the regime (as they were in Section 3), but also when the regime can undertake a preventive purge.

even more ferocious during the early years of the Cultural Revolution, 1966–76, which saw a combination of elite purges by Mao's faction and mass terror (MacFarquhar and Schoenhals, 2006).

¹⁴Our treatment does not exclude the possibility that there are multiple optimal levels of r^* . In that case, the results should be stated using the *strong set order* (Milgrom and Shannon, 1994).

4.1 Eliminating Diversity

We are now ready to examine how diversity affects propaganda. The main result of this section establishes that propaganda is more intense in less diverse societies.

Proposition 3. *Let f_1 and f_2 be two densities that satisfy Assumption 1, and suppose $r \leq \mu_s$. If f_2 is less diverse than f_1 , then the level of propaganda is higher under f_2 than under f_1 , that is, $\beta_2^*(r) \geq \beta_1^*(r)$.*

To gain some intuition, recall that the leader's propaganda level targets a marginal citizen who, upon receiving the positive message, barely stays home. Citizens of lower beliefs protest against the leader. The optimal strategy balances the autocrat's goal of minimizing the mass of protesters with that of minimizing the likelihood of protest. In a less diverse society, the beliefs are tightly concentrated around the modal citizen's belief. Therefore, targeting a citizen whose belief is slightly above the mode ensures that almost everyone stays home. But in a diverse society, this same approach yields too many protesters. To counter this, the autocrat needs to increase the credibility of propaganda, appealing to those with beliefs further from the mode.

Proposition 3 requires $r \leq \mu_s$, that is, the repression level should not be too high compared to the leader's confidence in his competence. Having $r \leq \mu_s$ allows us to compare two societies' diversity levels only through f as we did in Definition 1. If $r > \mu_s$, it is still possible to define a measure of diversity, but we need to impose a joint condition on f , r , and μ_s . We do this in Appendix B.

Throughout the analysis, we considered the distribution of opinions to be exogenous and remained agnostic about the forces that may increase diversity. Two channels that may lead to increased diversity are independent media and online media. In a recent working paper, [Enikolopov, Rochlitz, Schoors and Zakharov \(2025\)](#) demonstrate that access to independent online TV in Russia before the 2016 elections had asymmetric effects on individuals who rely on news from social media. Specifically, it boosted support among supporters of the regime, leading to a decline in support among those who opposed the regime. Motivated by their findings, and in light of the discussion here, one can argue that online media not only affect the attitudes of citizens but also have an impact on the effectiveness of traditional state-controlled media. In particular, online media do not have to convince every citizen—as long as they influence the opinions of *some* citizens, they could reduce the leader's incentives to engage in propaganda.

In Section 5, where we discuss the robustness of our results, we use the results of [Kolotilin, Mylovanov and Zapechelnyuk \(2022\)](#) to demonstrate that Proposition 3 naturally extends to the continuous action space. Proposition 6 makes this formal.

4.2 Eliminating Opponents

An important feature of our model of repression so far is that the punishment was only applied to those who actually acted against the leader. Now, we consider another type of repression, in which citizens are targeted *ex ante*, rather than *ex post*. For instance, the leader might identify, perhaps with some noise, those who are hostile to the regime and eliminate them. In other words, the diversity of the society is no longer fixed, the leader can now reduce the diversity of opinions in society and increase conformity. Repression alters the distribution of attitudes towards the regime, rather than the citizens' incentive to protest. We refer to such strategies as *ex ante repression*. Our analysis below demonstrates that the effects of *ex ante* repression are qualitatively the same: The more “pro-regime” or “conformist” the resulting distribution, the higher the level of propaganda for those who are not eliminated.

Consider a leader who can purge his opponents at will. Let f_1 be an initial density of priors that satisfies Assumption 1. The leader has access to some informative signal about the citizens' priors. Suppose that there is an institution, for example, a secret police, that assigns a label $\ell_i \in \{Skeptical, Supporter\}$ to each citizen i with prior μ_i . The secret police's labeling technology is noisy:

$$\begin{aligned}\Pr(\ell_i = Supporter \mid \mu_i) &= \rho(\mu_i), \\ \Pr(\ell_i = Skeptical \mid \mu_i) &= 1 - \rho(\mu_i),\end{aligned}$$

where $\rho(\mu_i) \in [0, 1]$. Suppose $\rho(\mu)$ is continuously differentiable, increasing, and log-concave in $\mu \in [0, 1]$. This formulation allows for a variety of labeling technologies, such as the linear ($\rho(\mu) = \mu$) or sigmoid ($\rho(\mu) = \frac{1}{1+e^{-\mu}}$) functions.

It is straightforward to see that the leader prefers to purge only those who are labeled *Skeptical*. Suppose for simplicity that the leader purges all citizens labeled $\ell_i = Skeptical$ from society and only those with the label $\ell_i = Supporter$ remain. Denote the distribution of priors for the remaining citizens by f_2 :

$$f_2(\mu) = \frac{\rho(\mu)f_1(\mu)}{\int_0^1 \rho(\tilde{\mu})f_1(\tilde{\mu})d\tilde{\mu}}.$$

As long as $\rho(\mu)$ is continuously differentiable and log-concave, $f_2(\mu)$ satisfies Assumption 1.¹⁵ Moreover,

$$\frac{f_2(\mu)}{f_1(\mu)} = \frac{\rho(\mu)}{\int_0^1 \rho(\tilde{\mu}) f_1(\tilde{\mu}) d\tilde{\mu}},$$

which is increasing in μ because $\rho(\mu)$ is increasing. That is, f_2 is larger than f_1 in the *likelihood ratio order*.

The following proposition shows that the leader engages in more propaganda following a purge of skeptics:

Proposition 4. *Consider two distributions of priors, f_1 and f_2 , which both satisfy Assumption 1 and where f_2 is larger than f_1 in the likelihood ratio order:*

$$\frac{f_2(\mu)}{f_1(\mu)} \text{ is increasing in } \mu.$$

Then, the propaganda level under f_2 is larger than the propaganda level under f_1 , that is, for any $r \in (0, 1)$,

$$\beta_2^*(r) \geq \beta_1^*(r).$$

Proposition 4 shows that *ex ante* repression and propaganda complement each other as well. With skeptics purged, the remaining citizens face more intense propaganda. Intuitively, the leader facing the distribution f_2 is a “more universally loved” leader than a leader facing the distribution f_1 . In equilibrium, a more popular leader *rides on his popularity* and provides less information to citizens, resulting in a higher level of information manipulation.

5 Robustness and Discussion

In this section, we discuss alternatives to our main assumptions and the robustness of our main results. Specifically, we focus on the assumptions of full commitment on behalf of the sender, the binary action space, and heterogeneous priors. In each case, our comparative statics results are robust to alternative assumptions. The choice of assumptions for the main model is dictated by our desire to keep the model as tractable as possible and by the real-world context we analyze.

¹⁵Most importantly, $\log f_2(\mu) = \log \rho(\mu) + \log f_1(\mu) - \log \int_0^1 \rho(\tilde{\mu}) f_1(\tilde{\mu}) d\tilde{\mu}$, which is strictly concave in μ , and therefore, $f_2(\mu)$ is strictly log-concave.

5.1 Partial Commitment

The assumption of a full commitment to information design by the sender (the leader) simplifies the technical analysis, but it is not necessary. Consider a more general model that does not assume full commitment on the sender's part. Let us introduce the probability p , $0 \leq p \leq 1$ and give the leader the opportunity to manipulate the outcome *ex post* with probability p . In other words, we allow the commitment to information design to fail with some probability. The $p = 0$ case corresponds to the model studied so far, a Bayesian persuasion model. When $p > 0$, we have a more general model of information manipulation without full commitment. The $p = 1$ case corresponds to the Crawford-Sobel “no commitment” communication protocol (Crawford and Sobel, 1982).

To make the point that our qualitative insights extend to this model, we fix a distribution of beliefs f and a repression level r . The leader's benefit from inducing posterior μ for a citizen i with the same prior as the leader (i.e., a citizen i with $\mu_i = \mu_s$) is $v(\mu; r)$ defined in (5). Suppose the leader chooses an information strategy with two messages, $m \in \{good, bad\}$, with

$$\begin{aligned}\sigma(m = good|\omega = competent) &= 1, \\ \sigma(m = good|\omega = incompetent) &= \widehat{\beta}_p(r) \in [0, 1].\end{aligned}$$

Given Theorem 1 of Lipnowski, Ravid and Shishkin (2022) and Lemma 1, the sender-optimal equilibrium of the game with partial commitment indeed contains such an information strategy. In equilibrium, the message is drawn according to σ , and whenever the leader has the opportunity to manipulate the message, he manipulates it so that citizens observe $m = good$. Therefore, the ex ante probability that the leader assigns to the citizens observing $m = good$ is

$$(1 - p) \left(\mu_s + (1 - \mu_s) \widehat{\beta}_p(r) \right) + p,$$

and with the complementary probability, citizens observe $m = bad$. Citizen i 's posterior that $\omega = competent$ after observing $m = good$ is

$$\frac{\mu_s}{(1 - p) \left(\mu_s + (1 - \mu_s) \widehat{\beta}_p(r) \right) + p},$$

and the posterior that $\omega = competent$ after observing $m = bad$ is zero. Therefore, the leader's

chosen propaganda level solves the following optimization problem:

$$\widehat{\beta}_p(r) = \arg \max_{\beta \in [0,1]} (1 - \mu_s)(1 - \beta)(1 - p) \cdot v(0; r) + ((1 - p)(\mu_s + (1 - \mu_s)\beta) + p) \cdot v\left(\frac{\mu_s}{(1 - p)(\mu_s + (1 - \mu_s)\beta) + p}; r\right). \quad (7)$$

Note that the objective function is a modified version of (6), and the two functions coincide when $p = 0$.

Our next result characterizes the propaganda level chosen under partial commitment in relation to the full-commitment propaganda level.

Proposition 5. *Given $p \in [0, 1]$, the propaganda level $\widehat{\beta}_p(r)$ satisfies:*

$$\widehat{\beta}_p(r) = \max \left\{ \frac{\beta^*(r) - p}{1 - p}, 0 \right\}.$$

An implication of Proposition 5 is that $\widehat{\beta}_p(r) \leq \beta^*(r)$ for any p . As in the running example of [Lipnowski, Ravid and Shishkin \(2022\)](#), the leader commits to a more informative structure (less propaganda) to compensate for the fact that the beliefs following $m = \text{good}$ will be distorted downwards because the citizens realize that the leader may have manipulated the message.

For our purposes, Proposition 5 reveals that the comparative statics results under full commitment (Propositions 1, 4 and 3) carry over to the case of partial commitment. That is, even with partial commitment, the level of propaganda is increasing in repression and decreasing in diversity.

5.2 Non-Binary Action Space

Our main motivation for the use of the binary action space comes from political economy considerations. Although the leaders of a country might have various motives, the standard assumption that a leader maximizes his chances of staying in power allows us to consider democratic and authoritarian leaders within the same analytical framework. For an authoritarian leader, this means preventing a coup, a revolution, or a massive protest that would lead to an ouster. ([Svolik, 2012](#) provides statistics that these types of exit cover the overwhelming majority of autocrats' exits; see [Dorsch and Maarek, 2018](#), and [Egorov and Sonin, 2024](#), for updated statistics.) Following the early formal theories of autocrats' critical moments ([Kuran, 1989](#); [Lohmann, 1993](#)), nearly all modern models assume a binary choice, "support" ("abstain") vs. "no support" ("rebel"), for citizens ([Persson and Tabellini, 2009](#); [Bueno de Mesquita, 2010](#);

Shadmehr and Bernhardt, 2011; Little, 2012; Edmond, 2013; Tyson and Smith, 2018; Shadmehr, 2019; Barbera and Jackson, 2020; Egorov and Sonin, 2021).

Although the binary action space for citizens is a standard assumption in models of authoritarian control, it greatly simplifies our analysis of persuasion. Specifically, the proof of Proposition 7 that deals with private vs. public persuasion does not readily extend to a larger action space. (See also a discussion of the binary vs. non-binary action space in Kolotilin et al., 2017.) Still, Propositions 1–4 on the complementarity between propaganda and repression would continue to hold even if we used a larger action space. As long as propaganda shifts citizens' actions toward those favored by the leader, repression will have a complementary effect by enhancing the incentives to take those preferred actions.

5.3 Heterogeneous Priors vs. Heterogeneous Payoffs

The choice of heterogeneous priors in our model is guided by the goal of being as realistic as possible. As discussed above, in an authoritarian society, a change at the top requires collective action by citizens, which justifies the assumption of a binary action space.

From a theoretical standpoint, assuming heterogeneous priors with a binary action space is almost equivalent to assuming heterogeneous payoff functions for citizens. There is still a subtle difference; as with heterogeneous priors, one cannot use the garbling result of Blackwell, 1953, as Kolotilin et al. (2017) does, so we rely on an appropriate modification in Section 5.5. In the next subsection, we present a version of the model featuring heterogeneous preferences rather than heterogeneous priors, and we show that our main insights carry over.

With heterogeneous payoffs, citizens have the same priors about the leader's competence, yet have different individual payoffs when the leader is competent. With heterogeneous priors, the payoffs are the same, but the subjective probabilities about the leader's competence differ. The empirical literature on authoritarian transitions points to the heterogeneity of people's beliefs about the leader's quality (e.g., Kuran, 1989). In a recent study on the impact of propaganda amid increased repressions on the Russian attitude towards the Russia-Ukraine war, Alyukov (2022) notes the heterogeneity of attitudes.

5.4 Continuous State Space

The main reason we opted for a binary state space, rather than a continuous one, is tractability. Suppose that the leader's type is instead distributed over $\Omega = [0, 1]$ with some continuous

density. The belief of each citizen $i \in I$ is a distribution $F_i \in \Delta(\Omega)$, and a distribution of priors is an object in $\Delta(\Delta(\Omega))$. Generalizing our results to such a setting requires defining a measure of diversity as in Definition 1 for distributions over $\Delta(\Delta(\Omega))$, a much more difficult task. With a binary state, each citizen is represented by a single number μ_i ; thus, there is a natural order of citizens based on their priors and a simple transformation of the distribution of priors that captures the idea of less diversity.

However, one can adopt a setting with heterogeneous *preferences* and a continuous state space as follows: There is a state of the world, denoted by $\omega \in \Omega = [0, 1]$. Here, ω captures the leader's competence, with higher numbers corresponding to more competent leaders. Citizens and the autocrat share a common prior that ω is distributed with a density $g(\omega)$. Citizen i 's payoff from staying home is normalized to zero. The cost of protesting against the leader to the citizen i is $-z_i - r$ if the leader is competent and $1 - z_i - r$ otherwise, where $z_i \in (-r, 1 - r)$ is privately observed by the citizen i . Citizens are heterogeneous in their support costs, with $f(z_i)$ denoting the density of z_i 's. We assume that z_i is the private information of citizen i and that f is common knowledge and continuously differentiable and bounded over its support.

Without loss of generality, assume that $M = [0, 1]$ and messages are *direct*, i.e., $\mu = \mathbb{E}[\omega | \mu]$ for each $\mu \in M$. Then, following a message $\mu \in M$, citizen i protests if and only if $z_i \leq 1 - \mu - r$. We conclude that the leader's value function is as follows:

$$v(\mu; r) = -F(1 - \mu - r).$$

Suppose f is strictly quasi-concave. Then, the value function is strictly S-shaped. By Theorem 1 of [Kolotilin, Mylovanov and Zapechelnyuk \(2022\)](#), for any g , the unique optimal strategy is an upper censorship. Here, an *upper censorship* strategy with cutoff $\omega^* \in [0, 1]$ reveals the states below ω^* and pools the states above ω^* . That is, under the optimal strategy, the leader reveals his competence level if his competence is below a certain threshold; otherwise, the leader censors all news about his competence level. Consequently, a low value of ω^* can be interpreted as a more intense (less informative) propaganda level.

In this setting, by Proposition 2 of [Kolotilin, Mylovanov and Zapechelnyuk \(2022\)](#), we directly conclude that ω^* decreases in r . That is, as in Proposition 1, higher repression leads to more propaganda, and hence our insights in Section 3 continue to hold. Regarding the relationship between diversity and propaganda, we have the following generalization of Proposition 3:

Proposition 6. *Let f_1 and f_2 be two single-peaked densities. If f_1 is less diverse than f_2 , then the level of propaganda is higher under f_2 than f_1 : $\omega_2^* \leq \omega_1^*$.*

Proposition 6 reveals that in a model with a continuum of states the diversity of preferences can still act as a bulwark against autocratic propaganda.

5.5 Targeted Propaganda: Public vs. Private Persuasion

In the analysis of repression in Section 4, the crucial precondition for the complementarity of propaganda and repression was the reliance on purges being *targeted*. A natural question to ask is whether the leader also benefits from targeted information manipulation. In this section, we argue that the answer is, again, determined by the ability of the leader to determine the citizens' beliefs towards him. If the leader knows the private beliefs of citizens, then he can benefit from this knowledge, targeting information that citizens receive—provided that citizens do not have access to any other sources.¹⁶

However, if the leader has to rely on citizens' self-selection into a unique source of information and design type-specific propaganda—that is, allowing for *private persuasion*—then the leader's toolkit is not actually expanded. We show that it is never (strictly) optimal for the leader to create different information sources that appeal to different citizens. Therefore, under the optimal policy, the leader continues to use public propaganda, supplemented by repression as in Section 3.

The first observation is almost trivial from a theoretical point of view: If the leader can address each citizen individually, then he can achieve the maximum possible level of persuasion and ultimately maximize his support. Critically, this requires the leader to keep each citizen in the dark about the messages that other citizens receive. However, sending individualized messages to citizens would require the leader to know each individual's belief. In authoritarian regimes, dissidents have a strong incentive to conceal their type out of fear of being singled out for repression. The leader then may need to rely on each citizen to self-select into receiving the message intended for her type. To analyze such a situation, we need to introduce the formalism of *incentive compatible persuasion* (Kolotilin et al., 2017; Bergemann and Morris, 2019).

Consider the setup in Section 2, but suppose that instead of being restricted to use a public

¹⁶If citizens have access to other sources of information, we are again in the realm of public persuasion previously analyzed. Restriction to a public persuasion mechanism is without loss of generality regardless of the number of sources freely accessible to citizens. To see this, suppose there are multiple information sources $1, \dots, n$ with message spaces M_1, \dots, M_n and information structures $\{\sigma_j(\cdot|\omega)\}_{\omega \in \{0,1\}} \in \Delta(M_j)$, $j = 1, \dots, n$. As long as citizens can observe messages from various sources, one can define $M \equiv M_1 \times \dots \times M_n$, and, for each $m = (m_1, \dots, m_n) \in M$, let $\sigma(m|\omega) = \sigma_1(m_1|\omega) \cdot \dots \cdot \sigma_n(m_n|\omega)$ for all $\omega \in \{0, 1\}$, so that the same outcome can be implemented via a public persuasion mechanism.

persuasion mechanism, the leader can use a *private persuasion mechanism* as follows. Each citizen reports her prior $\tilde{\mu} \in [0, 1]$ to the mechanism. The mechanism then sends an action recommendation $m_i \in \{p, s\}$ to citizen i , where $m_i = p$ stands for recommending $a_i = \textit{protest}$ and $m_i = s$ stands for recommending $a_i = \textit{stay}$.¹⁷

Definition 2. A **persuasion mechanism** σ is $\sigma = \{\sigma(\tilde{\mu}, \omega)\}_{\tilde{\mu} \in [0,1], \omega \in \{\textit{incompetent}, \textit{competent}\}}$, where

$$\sigma(\tilde{\mu}, \omega) = \Pr(m = p | \tilde{\mu}, \omega) \in [0, 1] \quad \text{for } \tilde{\mu} \in [0, 1], \omega \in \{\textit{incompetent}, \textit{competent}\}.$$

Consider a receiver i with prior $\mu_i = \mu$. Her (subjective) payoff from reporting a prior $\tilde{\mu}$, taking action $d_p \in \{\textit{protest}, \textit{stay}\}$ following message $m = p$, and taking action $d_s \in \{\textit{protest}, \textit{stay}\}$ following $m = s$ is

$$\begin{aligned} U_\sigma(\mu, \tilde{\mu}, d_p, d_s) \equiv & \mu \left(\sigma(\tilde{\mu}, \textit{competent}) \mathbb{1}_{d_p=\textit{protest}} + (1 - \sigma(\tilde{\mu}, \textit{competent})) \mathbb{1}_{d_s=\textit{protest}} \right) (-r) \\ & + (1 - \mu) \left(\sigma(\tilde{\mu}, \textit{incompetent}) \mathbb{1}_{d_p=\textit{protest}} + (1 - \sigma(\tilde{\mu}, \textit{incompetent})) \mathbb{1}_{d_s=\textit{protest}} \right) (1 - r). \end{aligned} \quad (8)$$

Incentive compatibility is defined in the usual way.

Definition 3. A persuasion mechanism σ is **incentive compatible** if, for all $\mu \in [0, 1]$,

$$U_\sigma(\mu, \mu, \textit{protest}, \textit{stay}) \geq U_\sigma(\mu, \tilde{\mu}, d_p, d_s) \quad \text{for all } \tilde{\mu} \in [0, 1], d_p, d_s \in \{\textit{protest}, \textit{stay}\}.$$

Incentive compatibility requires it to be a best response for citizens to be *truthful* and *obedient*. In particular, in an incentive compatible mechanism, no citizen could benefit from misreporting her (belief) type or deviating from the action recommended by the mechanism. If σ is an incentive compatible private persuasion mechanism, citizen i voluntarily chooses to receive messages designed exclusively for her, drawn according to $\sigma(\mu_i, \omega)$.

The assumption that each citizen follows only one news source requires an explanation of its own. One standard explanation is cognitive constraints. Another explanation is a budget constraint in terms of opportunity costs. If citizens could follow more than one source at no cost, then, of course, they would follow all available news sources. In this case, any persuasion mechanism would be equivalent to a public persuasion mechanism, which we analyzed in Section 3.

Let $U_\sigma(\mu)$ denote the subjective reward of a citizen with a prior μ who is truthful and obedient

¹⁷The sufficiency of two messages is immediate from a revelation principle argument.

(i.e., follows the news source designed for her and takes the recommended action). We have

$$\begin{aligned} U_\sigma(\mu) &\equiv U_\sigma(\mu, \mu, \text{protest}, \text{stay}) \\ &= \mu \cdot \sigma(\mu, \text{competent}) \cdot (-r) + (1 - \mu) \cdot \sigma(\mu, \text{incompetent}) \cdot (1 - r). \end{aligned} \quad (9)$$

Standard mechanism design arguments yield the following result:

Lemma 2. *If σ is an incentive compatible persuasion mechanism, then $U_\sigma(\mu)$ is convex, with $U_\sigma(0) = 1 - r$ and $U_\sigma(1) = 0$. Moreover, both $\sigma(\mu, \text{incompetent})$ and $\sigma(\mu, \text{competent})$ are decreasing in μ , with*

$$U'_\sigma(\mu) = \sigma(\mu, \text{competent})(-r) - \sigma(\mu, \text{incompetent})(1 - r) \quad (10)$$

for all $\mu \in [0, 1]$.

We will prove that for any incentive compatible private persuasion mechanism, there exists a direct public mechanism that achieves the same outcome. Formally, a public persuasion mechanism $\pi : \{0, 1\} \rightarrow \Delta(M)$ is a distribution of messages in each state. Let $M = [0, 1]$, and consider direct public mechanisms for the sender: for each $\mu \in M$, $\Pr_s(\omega = \text{competent} | \mu) = \mu$. That is, under a direct public mechanism, the sender's posterior following a message is the message itself.¹⁸ Given a state ω , let $\pi(\mu | \omega)$ denote the cdf of messages. Consequently, the ex ante distribution of messages (from the sender's perspective) is $\pi_s(\mu) = \mu_s \pi(\mu | \omega = \text{competent}) + (1 - \mu_s) \pi(\mu | \omega = \text{incompetent})$. For a mechanism to be direct, it needs to satisfy $\mathbb{E}_s[\mu] = \mu_s$.

We say that a direct public mechanism π **achieves the same outcome** as a private persuasion mechanism σ if it induces the same distribution of actions for each receiver in each state. Note that because of the heterogeneity of priors, citizens expect different states to be realized with different probabilities, so the *ex ante* distributions of actions still differ.

Proposition 7. *For any incentive compatible private persuasion mechanism σ , there exists a direct public mechanism π that achieves the same outcome.*

Proposition 7 implies that the leader can achieve any result through a public persuasion mechanism, where he offers the same information structure to each citizen. This justifies our focus on public mechanisms in Section 3 and implies that the optimal policy we characterized yields a higher payoff to the leader than any incentive compatible private persuasion

¹⁸Because we are working with heterogeneous priors, the posterior following a message is different across receivers. Therefore, it is impossible to define a *direct* public mechanism for everyone. We opt for defining a direct public mechanism for a particular agent, the sender.

mechanism. Intuitively, this is because the incentive compatibility constraints are extremely tight for the leader, to the extent that a public mechanism (which trivially satisfies incentive compatibility) can yield the same payoff.

This result is closely related to the “impossibility of private persuasion” result in [Kolotilin et al. \(2017\)](#); the difference is that our result is in a setup with heterogeneous priors, rather than with heterogeneous preferences. We cannot rely on the garbling result of [Blackwell \(1953\)](#) as [Kolotilin et al. \(2017\)](#) do, as Blackwell’s famed result requires common posteriors following a public message. However, in our proof, we modify the proof of Proposition 2 in the Online Appendix to [Kolotilin et al. \(2017\)](#) to obtain a similar result in our setting.

Proposition 7 provides a rationale for why authoritarian regimes might prefer a standardized approach to censorship and mass propaganda. The reality is, of course, more complex. Vladimir Lenin, the founder of the communist state after the Russian Revolution, stated in 1921: “We need full and truthful information. And the truth should not depend on who it has to serve. We can accept only the division between the unofficial information (for the Comintern Executive Committee only) and official information (for everybody).”¹⁹ In a famous example of information presented differently to different audiences, the 1956 speech by Soviet leader Nikita Khrushchev denouncing Stalin’s repressions and cult of personality was distributed, in written form, to local communist party secretaries, read aloud to ordinary party members at specially arranged meetings, and kept secret from the rest of the population ([Taubman, 2003](#)). In effect, different groups of citizens were given access to information tailored to their classification. Critically, the Soviet leader relied on the existing designation of citizen types, with the communist party members presumably being more loyal than citizens at large. Proposition 7 suggests that, absent the existing division of citizens into observable types, Khrushchev could not do better than to rely on public persuasion.

6 Conclusion

In this paper, we offer a model of information manipulation and repression, the two main tools in any autocrat’s arsenal. We consider both public and private persuasion and different types of repression. With a higher level of repression, the leader’s marginal supporter becomes more predisposed to support him and is thus more susceptible to manipulation. A more

¹⁹The Lenin’s formula was employed by other communist regimes as well; see [Gehlbach and Keefer \(2011\)](#), [King, Pan and Roberts \(2013\)](#) and [Lorentzen \(2014\)](#) for the analysis of China’s information policy.

diverse society poses a problem for an authoritarian regime as information manipulation is less effective in such circumstances, so repressions that reduce diversity have an additional impact.

In George Orwell's totalitarian dystopia, citizens of Oceania are forced to use the *newspeak*, a special language designed to limit their ability to articulate antigovernment concepts, cannot switch off the radio that transmits propaganda, and are forced to participate in ideological indoctrination meetings. However, the ultimate message of *1984* is that it is physical torture, applied to some, that makes citizens believe what the government wants them to believe.

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Appendices

A Proofs

Proof of Lemma 1. As f is continuously differentiable by Assumption 1, $v''(\mu; r) = \frac{\partial^2 v(\mu; r)}{\partial \mu^2}$ exists. We will show that $v''(\mu; r)$ satisfies the *strict single-crossing-from-above property*:

If $v''(\mu_1; r) \geq 0$ for some $\mu_1 \in [0, 1]$, then $v''(\mu_2; r) > 0$ for all $\mu_2 < \mu_1$,

which implies the result.

By (5), $v(\mu; r) = -F(g(\mu))$ where:

$$g(\mu) \equiv \frac{1 - \mu}{1 - \mu + \gamma \mu}, \quad \gamma \equiv \frac{r}{1 - r} \frac{1 - \mu_s}{\mu_s} > 0.$$

Then,

$$v''(\mu; r) = -f'(g(\mu)) \cdot (g'(\mu))^2 - f(g(\mu)) \cdot g''(\mu).$$

Suppose $v''(\mu_1; r) \geq 0$ for some $\mu_1 \in [0, 1]$. This implies

$$\frac{f'(g(\mu_1))}{f(g(\mu_1))} \leq -\frac{g''(\mu_1)}{(g'(\mu_1))^2} = \frac{2}{\gamma} \left((1 - \gamma) - (1 - \gamma)^2 \mu_1 \right). \quad (11)$$

Take any $\mu_2 < \mu_1$. Because $g(\mu)$ is strictly decreasing in μ , $g(\mu_2) > g(\mu_1)$. Because $f(\mu)$ is strictly log-concave by Assumption 1,

$$\frac{f'(g(\mu_2))}{f(g(\mu_2))} < \frac{f'(g(\mu_1))}{f(g(\mu_1))}. \quad (12)$$

Moreover, $((1 - \gamma) - (1 - \gamma)^2 \mu)$ is decreasing in μ , so equations (11)–(12) yield that

$$\frac{f'(g(\mu_2))}{f(g(\mu_2))} < \frac{2}{\gamma} \left((1 - \gamma) - (1 - \gamma)^2 \mu_2 \right) = -\frac{g''(\mu_2)}{(g'(\mu_2))^2}$$

Therefore, $v''(\mu_2; r) > 0$. □

Proof of Proposition 1. Consider $0 < r_1 \leq r_2 < 1$. For $k \in \{1, 2\}$, the value function under r_k is $v(\mu; r_k)$. By Lemma 1, $v(\mu; r_k)$ is strictly S-shaped. By Corollary 2 of [Kamenica and Gentzkow \(2011\)](#), the optimal policy generates two posteriors for the leader: $\mu \in \{0, \hat{\mu}_k\}$. Here, $\hat{\mu}_k$ solves²⁰

$$\hat{\mu}_k = \arg \max_{\mu \in [\mu_s, 1]} \frac{1 + v(\mu; r_k)}{\mu} = \arg \max_{\mu \in [\mu_s, 1]} \frac{1 - F\left(\frac{1 - \mu}{1 - \mu + \gamma_k \mu}\right)}{\mu}, \quad (13)$$

²⁰The sender's problem is choosing a pair of posteriors $\{0, \mu\}$. By Bayes plausibility, $\mu \geq \mu_s$ and μ will be realized with probability $\frac{\mu_s}{\mu}$. Then, the sender chooses μ to maximize $(1 - \frac{\mu_s}{\mu})v(0; r_k) + \frac{\mu_s}{\mu}v(\mu; r_k) = (1 - \frac{\mu_s}{\mu})(-1) + \frac{\mu_s}{\mu}v(\mu; r_k)$. Removing the terms that do not affect the maximizer yields (13).

where $\gamma_k = \frac{r_k}{1-r_k} \frac{1-\mu_s}{\mu_s}$. Strict S-shape of $v(\mu; r_k)$ ensures that there is a unique solution to this problem, i.e., the objective function is strictly quasi-concave.

Adopting the following change of variables:

$$z = \frac{1-\mu}{1-\mu+\gamma_k\mu} \iff \mu = \frac{1-z}{1-z+\gamma_k z},$$

we see that $\hat{\mu}_k = \frac{1-\hat{z}_k}{1-\hat{z}_k+\gamma\hat{z}_k}$, where

$$\hat{z}_k = \arg \max_{z \in [0, 1-r_k]} \frac{1-F(z)}{1-z} (1-z+\gamma_k z).$$

Since z is strictly decreasing in μ , strict quasi-concavity of the objective function in μ implies strict quasi-concavity of $\frac{1-F(z)}{1-z} (1-z+\gamma_k z)$ in z . Therefore, \hat{z}_k satisfies the first-order condition:

$$\frac{d}{dz} \left(\frac{1-F(z)}{1-z} (1-z+\gamma_k z) \right) \geq 0 \iff z \leq \hat{z}_k. \quad (14)$$

Noting that

$$\begin{aligned} & \frac{d}{dz} \left(\frac{(1-F(z))(1-z+\gamma_k z)}{1-z} \right) \\ &= \frac{(-f(z)(1-z+\gamma_k z) + (1-F(z))(-1+\gamma_k)) (1-z) + (1-F(z))(1-z+\gamma_k z)}{(1-z)^2} \\ &= \frac{-f(z)(1-z+\gamma_k z)(1-z) + (1-F(z))\gamma_k}{(1-z)^2}, \end{aligned}$$

we have:

$$\begin{aligned} \frac{d}{dz} \left(\frac{1-F(z)}{1-z} (1-z+\gamma_k z) \right) \geq 0 &\iff -f(z)(1-z+\gamma_k z)(1-z) + (1-F(z))\gamma_k \geq 0 \\ &\iff \frac{1-F(z)}{1-z} \geq f(z) \frac{1-z+\gamma_k z}{\gamma_k}. \end{aligned}$$

Thus, condition (14) becomes:

$$\frac{1-F(z)}{1-z} \geq f(z) \frac{1-z+\gamma_k z}{\gamma_k} \iff z \leq \hat{z}_k. \quad (15)$$

Recall that $r_1 \leq r_2$, and hence $\gamma_1 \leq \gamma_2$. Consider any $z \leq \hat{z}_1$. Then,

$$\frac{1-F(z)}{1-z} \underbrace{\geq}_{(15)} f(z) \frac{1-z+\gamma_1 z}{\gamma_1} \underbrace{\geq}_{\gamma_1 \leq \gamma_2} f(z) \frac{1-z+\gamma_2 z}{\gamma_2},$$

which, by (15), implies: $z \leq \hat{z}_2$. We conclude that $\hat{z}_1 \leq \hat{z}_2$. Since z is strictly decreasing in μ , $\hat{\mu}_1 \geq \hat{\mu}_2$.

To conclude the proof, consider three cases. If $\mu_s \geq \hat{\mu}_1$, the optimal policy does not reveal any information in either case. Given that we already set $\sigma(m = \text{good} | \omega = \text{competent}) = 1$, the

optimal policy includes $\sigma(m = \text{good}|\omega = \text{incompetent}) = 1$. Therefore, $\beta^*(r_1) = \beta^*(r_2) = 1$. If $\hat{\mu}_1 > \mu_s \geq \hat{\mu}_2$, the optimal policy under $\nu(\mu; r_2)$ does not reveal any information. In this case, $\beta^*(r_2) = 1$ and $\beta^*(r_1) < 1$. Finally, if $\mu_s < \hat{\mu}_2$, propaganda levels $\beta^*(r_1)$ and $\beta^*(r_2)$ satisfy

$$\frac{\mu_s}{\mu_s + (1 - \mu_s)\beta^*(r_k)} = \hat{\mu}_k, \quad \text{for } k \in \{1, 2\}.$$

Then, $\hat{\mu}_1 \geq \hat{\mu}_2$ implies $\beta^*(r_1) \leq \beta^*(r_2)$. \square

Proof of Proposition 3. As in the proof of Proposition 1, we show that $\hat{z}_1 \leq \hat{z}_2$. Consider some $z \leq \hat{z}_1$. Then,

$$\begin{aligned} \frac{1 - F_2(z)}{1 - z} &= \frac{\int_z^1 f_2(x) dx}{1 - z} = \frac{\int_z^1 \alpha(f_1(x)) dx}{1 - z} \underbrace{\geq}_{\text{Jensen's}} \alpha\left(\frac{\int_z^1 f_1(x) dx}{1 - z}\right) = \alpha\left(\frac{1 - F_1(z)}{1 - z}\right) \\ &\underbrace{\geq}_{(15)} \alpha\left(f_1(z) \frac{1 - z + \gamma z}{\gamma}\right) \underbrace{\geq}_{\text{Convexity of } \alpha(\cdot)} \alpha(f_1(z)) \frac{1 - z + \gamma z}{\gamma} \\ &= f_2(z) \frac{1 - z + \gamma z}{\gamma}, \end{aligned}$$

which, by (15), implies $z \leq \hat{z}_2$. Note that in the first inequality we use the integral form of Jensen's inequality (e.g., [Dragomir, Adil Khan and Abathun 2016](#)), and the last inequality relies on $r \leq \mu_s$ (which is equivalent to $\gamma \leq 1$). We conclude that $\hat{z}_1 \leq \hat{z}_2$, and thus $\hat{\mu}_1 \geq \hat{\mu}_2$. Replicating the argument in the Proof of Proposition 1 yields $\beta_1^*(r) \leq \beta_2^*(r)$. \square

Proof of Proposition 4. As in the proof of Proposition 1, we show that $\hat{z}_1 \leq \hat{z}_2$. Consider some $z \leq \hat{z}_1$. By (15),

$$\frac{1 - F_1(z)}{1 - z} \geq f_1(z) \frac{1 - z + \gamma z}{\gamma} \implies \frac{f_1(z)}{1 - F_1(z)} \leq \frac{\gamma}{(1 - z)(1 - z + \gamma z)}.$$

Because the likelihood ratio order implies the hazard rate order (see, e.g., Theorem 1.C.1 of [Shaked and Shanthikumar, 2007](#)), $\frac{f_1(x)}{1 - F_1(x)} \geq \frac{f_2(x)}{1 - F_2(x)}$ for all $x \in [0, 1]$, we have

$$\frac{f_2(z)}{1 - F_2(z)} \leq \frac{\gamma}{(1 - z)(1 - z + \gamma z)} \implies \frac{1 - F_2(z)}{1 - z} \geq f_2(z) \frac{1 - z + \gamma z}{\gamma},$$

which, by (15), implies $z \leq \hat{z}_2$. We conclude that $\hat{z}_1 \leq \hat{z}_2$, and thus $\hat{\mu}_1 \geq \hat{\mu}_2$. Replicating the argument in the Proof of Proposition 1 yields $\beta_1^*(r) \leq \beta_2^*(r)$. \square

Proof of Proposition 5. Consider the optimization problem in (7) with a change of variables, where:

$$\mu_p \equiv \frac{\mu_s}{(1 - p)(\mu_s + (1 - \mu_s)\beta) + p}. \quad (16)$$

Note that μ_p is strictly decreasing in β , with $\mu_p = \mu_s$ when $\beta = 1$, and $\mu_p = \frac{\mu_s}{(1-p)\mu_s+p}$ when $\beta = 0$. Therefore, $\mu_p \in [\mu_s, \frac{\mu_s}{(1-p)\mu_s+p}]$. With the change of variables, the optimization problem is

$$\max_{\mu_p \in [\mu_s, \frac{\mu_s}{(1-p)\mu_s+p}]} \left(1 - \frac{\mu_s}{\mu_p}\right) v(0; r) + \frac{\mu_s}{\mu_p} v(\mu_p; r). \quad (17)$$

Substituting $v(0; r) = -1$ and dropping the terms that do not affect the maximizer, we have:

$$\max_{\mu_p \in [\mu_s, \frac{\mu_s}{(1-p)\mu_s+p}]} \frac{1 + v(\mu_p; r)}{\mu_p}, \quad (18)$$

which is identical to the problem in (13), only with a different range of μ_p . Moreover, strict S-shape of $v(\mu; r)$ ensures that the objective function is strictly quasi-concave over the range of μ_p .

Let $\hat{\mu}_p$ denote the maximizer for problem (18), and let $\hat{\mu}$ denote the maximizer for the original problem under full commitment (13)—the one with $p = 0$. Note that, by (16),

$$\hat{\mu}_p = \frac{\mu_s}{(1-p) \left(\mu_s + (1-\mu_s) \widehat{\beta}_p(r) \right) + p}, \quad (19)$$

and, as shown at the end of the proof of Proposition 1,

$$\hat{\mu} = \frac{\mu_s}{\mu_s + (1-\mu_s) \beta^*(r)}. \quad (20)$$

To proceed with the proof, we consider three exhaustive cases:

1. If $\hat{\mu} = \mu_s$, the objective function $\frac{1+v(\mu; r)}{\mu}$ is decreasing in $\mu \in [\mu_s, 1]$, and hence it is also decreasing in $\mu \in [\mu_s, \frac{\mu_s}{(1-p)\mu_s+p}]$. Therefore, $\hat{\mu}_p = \mu_s$. By (19), $\widehat{\beta}_p(r) = 1$. By (20), $\beta^*(r) = 1$. Therefore, in this case, $\widehat{\beta}_p(r) = 1 = \frac{1-p}{1-p} = \frac{\beta^*(r)-p}{1-p} \geq 0$.
2. If $\hat{\mu} \in [\mu_s, \frac{\mu_s}{(1-p)\mu_s+p}]$, the maximizer of the objective function $\frac{1+v(\mu; r)}{\mu}$ remains in the relevant range of μ_p . Therefore, $\hat{\mu}_p = \hat{\mu}$. Combining this (19) and (20), we have:

$$\frac{\mu_s}{(1-p) \left(\mu_s + (1-\mu_s) \widehat{\beta}_p(r) \right) + p} = \frac{\mu_s}{\mu_s + (1-\mu_s) \beta^*(r)} \implies \widehat{\beta}_p(r) = \frac{\beta^*(r) - p}{1-p},$$

where, by the fact that $\hat{\mu} \leq \frac{\mu_s}{(1-p)\mu_s+p}$ and by (20), we have: $\beta^*(r) - p \geq 0$.

3. If $\hat{\mu} > \frac{\mu_s}{(1-p)\mu_s+p}$, the objective function $\frac{1+v(\mu; r)}{\mu}$ increases in the relevant range of μ_p . Therefore, $\hat{\mu}_p = \frac{\mu_s}{(1-p)\mu_s+p}$. By (19), $\widehat{\beta}_p(r) = 0$.

Moreover, $\hat{\mu} > \frac{\mu_s}{(1-p)\mu_s+p}$ and (20) imply that $\beta^*(r) < p$. Therefore, in this case, $\widehat{\beta}_p(r) = 0 \geq \frac{\beta^*(r)-p}{1-p}$.

In any case, we have shown that $\widehat{\beta}_p(r) = \max \left\{ \frac{\beta^*(r)-p}{1-p}, 0 \right\}$. \square

Proof of Proposition 6. Take two single-peaked densities f_1 and f_2 that satisfy equation (3). For $k \in \{1, 2\}$, define the leader's payoff from an upper-censorship strategy with cutoff ω^* as:

$$W_k(\omega^*) \equiv \int_0^{\omega^*} -F_k(1 - \omega - r)g(\omega)d\omega + \int_{\omega^*}^1 -F_k(1 - \mu^* - r)g(\omega)d\omega.$$

where

$$\mu^* \equiv \mathbb{E}[\omega | \omega \geq \omega^*] = \frac{\int_{\omega^*}^1 \omega g(\omega)d\omega}{1 - G(\omega)}$$

By Lemma 1 of [Kolotilin, Mylovanov and Zapechelnyuk \(2022\)](#), $W_k(\omega^*)$ is strictly quasi-concave. It can be derived (see, e.g., [Kolotilin, Mylovanov and Zapechelnyuk, 2022](#), p.565) that

$$W'_k(\omega^*) = g(\omega^*) (f_k(1 - \mu^* - r) (\mu^* - \omega^*) - (F_k(1 - \omega^* - r) - F_k(1 - \mu^* - r))). \quad (21)$$

We are now ready to prove that $\omega_1^* \geq \omega_2^*$. If $\omega_1^* = 1$, the inequality is satisfied. If $\omega_1^* < 1$, $W'_1(\omega_1^*) \leq 0$. By (21),

$$\frac{F_1(1 - \omega_1^* - r) - F_1(1 - \mu_1^* - r)}{\mu_1^* - \omega_1^*} \geq f_1(1 - \mu_1^* - r) \implies \frac{\int_{\omega_1^*}^{\mu_1^*} f_1(1 - \omega - r)d\omega}{\mu_1^* - \omega_1^*} \geq f_1(1 - \mu_1^* - r).$$

Then,

$$\begin{aligned} \frac{F_2(1 - \omega_1^* - r) - F_2(1 - \mu_1^* - r)}{\mu_1^* - \omega_1^*} &= \frac{\int_{\omega_1^*}^{\mu_1^*} f_2(1 - \omega - r)d\omega}{\mu_1^* - \omega_1^*} = \frac{\int_{\omega_1^*}^{\mu_1^*} \alpha (f_1(1 - \omega - r)) d\omega}{\mu_1^* - \omega_1^*} \\ &\stackrel{\text{Jensen's}}{\geq} \alpha \left(\frac{\int_{\omega_1^*}^{\mu_1^*} f_1(1 - \omega - r)d\omega}{\mu_1^* - \omega_1^*} \right) \\ &\geq \alpha (f_1(1 - \omega_1^* - r)) \\ &= f_2(1 - \omega_1^* - r). \end{aligned}$$

Therefore, by (21), $W'_2(\omega_1^*) \leq 0$. Since $W_2(\omega^*)$ is strictly quasi-concave, then, $W'_2(\omega^*) < 0$ for any $\omega^* > \omega_1^*$. We conclude that $\omega_2^* \leq \omega_1^*$. \square

Proof of Proposition 7. Consider the public persuasion mechanism defined as

$$\pi(\mu | \omega) = \sigma(g(\mu), \omega) \quad \text{for all } \mu \in [0, 1], \omega \in \{\text{incompetent}, \text{competent}\},$$

where

$$g(\mu) = \frac{1 - \mu}{1 - \mu + \gamma\mu}, \quad \gamma = \frac{r}{1 - r} \frac{1 - \mu_s}{\mu_s}.$$

Note that $g(\mu)$ is strictly decreasing in μ , with $g(0) = 1$ and $g(1) = 0$. Moreover, $g(g(\mu)) = \mu$ for all $\mu \in [0, 1]$.

Take $\mu, \mu' \in [0, 1]$ with $\mu \geq \mu'$. Since g is strictly decreasing, $g(\mu) \leq g(\mu')$. By Lemma 2, $\sigma(\mu, \omega)$ is decreasing in μ , which implies $\sigma(g(\mu), \omega) \geq \sigma(g(\mu'), \omega)$. Therefore, $\pi(\mu|\omega) \geq \pi(\mu'|\omega)$. We conclude that $\pi(\mu|\omega)$ is increasing in μ , and so, it is a cdf.

Next, we show that $\mathbb{E}_s[\mu] = \mu_s$, so that this is indeed a direct mechanism. By (9) and (10),

$$\sigma(\mu, \text{incompetent}) = \frac{U_\sigma(\mu) - \mu U'_\sigma(\mu)}{1 - r}. \quad (22)$$

Substituting (22) into (10), we obtain:

$$\sigma(\mu, \text{competent}) = -\frac{U_\sigma(\mu) + (1 - \mu)U'_\sigma(\mu)}{r}. \quad (23)$$

Then, we have:

$$\begin{aligned} \pi_s(\mu) &= \mu_s \pi(\mu|\omega = \text{competent}) + (1 - \mu_s) \pi(\mu|\omega = \text{incompetent}) \\ &= \mu_s \sigma(g(\mu), \text{competent}) + (1 - \mu_s) \sigma(g(\mu), \text{incompetent}) \\ &= -\frac{\mu_s}{r} \left[(1 - \gamma) U_\sigma(g(\mu)) + (1 - g(\mu) + \gamma g(\mu)) U'_\sigma(g(\mu)) \right] \quad (\text{by (22) and (23)}). \end{aligned}$$

Multiplying the above expression by $\frac{-\gamma}{(1 - g(\mu) + \gamma g(\mu))^2} g'(\mu) = 1$ and rearranging terms yields

$$\pi_s(\mu) = \frac{1 - \mu_s}{1 - r} \left[\frac{(1 - \gamma)}{(1 - g(\mu) + \gamma g(\mu))^2} U_\sigma(g(\mu)) + \frac{1}{1 - g(\mu) + \gamma g(\mu)} U'_\sigma(g(\mu)) \right] g'(\mu).$$

Taking the integral and changing variables with $v = g(\mu)$ gives

$$\int_0^1 \pi_s(\mu) d\mu = \frac{1 - \mu_s}{1 - r} \int_{v=g(0)}^{v=g(1)} \left[\frac{1 - \gamma}{(1 - v + \gamma v)^2} U_\sigma(v) + \frac{1}{1 - v + \gamma v} U'_\sigma(v) \right] dv.$$

Since $g(0) = 1$ and $g(1) = 0$, then

$$\begin{aligned} \int_0^1 \pi_s(\mu) d\mu &= -\frac{1 - \mu_s}{1 - r} \int_0^1 \left(\frac{d}{dv} \frac{U_\sigma(v)}{1 - v + \gamma v} \right) dv \\ &= -\frac{1 - \mu_s}{1 - r} \left(\frac{U_\sigma(1)}{\gamma} - U_\sigma(0) \right). \end{aligned}$$

By Lemma 2, $U_\sigma(0) = 1 - r$ and $U_\sigma(1) = 0$, so $\int_0^1 \pi_s(\mu) d\mu = 1 - \mu_s$. Therefore,

$$\mathbb{E}_s[\mu] = \mu_s,$$

and we conclude that the public persuasion mechanism π is direct.

Finally, we show that π induces the same distribution of actions as σ for each receiver in each state. This is true because, for each receiver i with prior μ_i and each state ω , the probability of

taking action $a_i = \textit{protest}$ is

$$\begin{aligned}
\Pr(a_i = \textit{protest}|\omega) &= \Pr\left(\frac{\mu \frac{\mu_i}{\mu_s}}{\mu \frac{\mu_i}{\mu_s} + (1 - \mu) \frac{1 - \mu_i}{1 - \mu_s}} \leq 1 - r \mid \omega\right) \\
&= \Pr\left(\mu \leq \frac{1 - \mu_i}{1 - \mu_i + \gamma \mu_i} \mid \omega\right) \\
&= \Pr(\mu \leq g(\mu_i) | \omega) \\
&= \pi(g(\mu_i) | \omega).
\end{aligned}$$

Since $\pi(\mu | \omega) = \sigma(g(\mu), \omega)$, $\Pr(a_i = \textit{protest} | \omega) = \sigma(g(g(\mu_i)), \omega)$. Moreover, since $g(g(\mu)) = \mu$ for each $\mu \in [0, 1]$, $\Pr(a_i = \textit{protest} | \omega) = \sigma(\mu_i, \omega)$, and the result follows. \square

B A General Measure of Diversity

In this section, we define a more general measure of diversity and show that the result of Proposition 3 holds even when $r > \mu_s$ under this more general measure.

Definition 4. Consider two log-concave distributions with densities f_1 and f_2 supported on a common compact set, and let $\gamma = \frac{r}{1-r} \frac{1-\mu_s}{\mu_s}$. f_2 is *generally less diverse* than f_1 given γ if

$$f_2(x) \frac{(1-x+\gamma x)^2}{\gamma} = \alpha \left(f_1(x) \frac{(1-x+\gamma x)^2}{\gamma} \right) \quad \text{for all } x \quad (24)$$

and some strictly increasing and convex function $\alpha : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ with $\alpha(0) = 0$.

The generally more diverse order is a generalization of the more diverse order defined in Definition 1. When $r = \mu_s$ (i.e. $\gamma = 1$), the two definitions are equivalent. The interpretation also carries the same flavor: An adjusted version of $f_2(x)$ is obtained by transforming an adjusted version of $f_1(x)$ through a convex function. The adjustment terms include r and μ_s . Thus, the definition 4 can be interpreted as f_2 being less diverse than f_1 after taking into account r and μ_s .

We are now ready to state and prove how Proposition 3 extends to the generally-less-diverse order.

Proposition 8. *Let f_1 and f_2 be two densities that satisfy Assumption 1. If f_2 is generally less diverse than f_1 , then the level of propaganda is higher under f_2 than under f_1 : $\beta_1^*(r) \leq \beta_2^*(r)$.*

Proof. As in the proof of Proposition 1, we show that $\hat{z}_1 \leq \hat{z}_2$. Consider some $z \leq \hat{z}_1$. Then,

$$\begin{aligned} \frac{1 - F_2(z)}{1 - z} (1 - z + \gamma z) &= \frac{\int_z^1 f_2(x) dx}{1 - z} (1 - z + \gamma z) \\ &= \frac{\int_z^1 f_2(x) dx}{\frac{1-z}{1-z+\gamma z}} \\ &= \frac{\int_z^1 \alpha \left(f_1(x) \frac{(1-x+\gamma x)^2}{\gamma} \right) \frac{\gamma}{(1-x+\gamma x)^2} dx}{\frac{1-z}{1-z+\gamma z}} && \text{(by (24))} \\ &= \frac{-\int_{\frac{1-z}{1-z+\gamma z}}^0 \alpha \left(f_1 \left(\frac{1-t}{1-t+\gamma t} \right) \frac{\gamma}{(1-t+\gamma t)^2} \right) dt}{\frac{1-z}{1-z+\gamma z}} && \text{(change of variables, } t = \frac{1-x}{1-x+\gamma x} \text{)} \\ &= \frac{\int_0^{\frac{1-z}{1-z+\gamma z}} \alpha \left(f_1 \left(\frac{1-t}{1-t+\gamma t} \right) \frac{\gamma}{(1-t+\gamma t)^2} \right) dt}{\frac{1-z}{1-z+\gamma z}} \end{aligned}$$

$$\begin{aligned}
&\geq \alpha \left(\frac{\int_0^{\frac{1-z}{1-z+\gamma z}} f_1 \left(\frac{1-t}{1-t+\gamma t} \right) \frac{\gamma}{(1-t+\gamma t)^2} dt}{\frac{1-z}{1-z+\gamma z}} \right) && \text{(by Jensen's Inequality)} \\
&= \alpha \left(\frac{-\int_1^z f_1(u) du}{\frac{1-z}{1-z+\gamma z}} \right) && \text{(change of variables, } u = \frac{1-t}{1-t+\gamma t} \text{)} \\
&= \alpha \left(\frac{\int_z^1 f_1(u) du}{1-z} (1-z+\gamma z) \right) \\
&= \alpha \left(\frac{1-F_1(z)}{1-z} (1-z+\gamma z) \right) \\
&\geq \alpha \left(f_1(z) \frac{1-z+\gamma z}{\gamma} (1-z+\gamma z) \right) && \text{(by } z \leq \hat{z}_1, \text{ (15), and increasing } \alpha \text{)} \\
&= \alpha \left(f_1(z) \frac{(1-z+\gamma z)^2}{\gamma} \right) \\
&= f_2(z) \frac{(1-z+\gamma z)^2}{\gamma} && \text{(by (24))}
\end{aligned}$$

Therefore,

$$\frac{1-F_2(z)}{1-z} (1-z+\gamma z) \geq f_2(z) \frac{(1-z+\gamma z)^2}{\gamma} \implies \frac{1-F_2(z)}{1-z} \geq f_2(z) \frac{1-z+\gamma z}{\gamma},$$

which, by (15), implies $z \leq \hat{z}_2$. We conclude that $\hat{z}_1 \leq \hat{z}_2$, and thus $\hat{\mu}_1 \geq \hat{\mu}_2$. Replicating the argument in the Proof of Proposition 1 yields $\beta_1^*(r) \leq \beta_2^*(r)$. \square