# Media Capture: A Bayesian Persuasion Approach *

Arda Gitmez † Pooya Molavi ‡

December 29, 2018

**Abstract**

We present a model of *media capture*, a politician having control over the editorial policies of media. At the heart of the model is the trade-off faced by a politician who wants to persuade the citizens: she wants to capture the media and produce news in her favor, but capture leads the citizens to not follow the media as they find them uninformative. The model is a Bayesian persuasion model (à la Kamenica and Gentzkow (2011)) with an audience of heterogeneous priors. We identify conditions on the distribution of priors that guarantee full information revelation and no information revelation by the captured media. The model also has several testable predictions: (i) the information content of the news provided by the captured media decreases as the politician becomes more popular, (ii) in societies with more extremists than moderates, the media are more likely to produce "negative" news than "positive" ones, and (iii) in societies where the media are less accessible to citizens, they are more informative.

*We would like to thank Daron Acemoglu, Mehmet Ekmekci, Drew Fudenberg, Emin Karagözoğlu, Parag Pathak, Iván Werning and Alex Wolitzky for helpful discussions, and the participants at MIT Theory Lunch and MIT Political Economy Lunch for their comments.

†Department of Economics, Massachusetts Institute of Technology. E-mail address: agitmez@mit.edu.

‡Department of Economics, Massachusetts Institute of Technology. E-mail address: mpooya@mit.edu.

# 1 Introduction

Modern democracies are increasingly distressed about *media capture*, a phenomenon where the politician has control over the editorial policies over a media source. However, economics literature has provided surprisingly few theoretical insights about media capture yet. The canonical model of the media capture, offered by Besley and Prat (2006), adopts a straightforward view about the way in which media operates. It assumes that the politician practices *tyranny* over the captured media sources and prevents them from sending any negative news about herself. According to *tyranny view*, if the media is captured, the citizens observe nothing but the positive news about the politician. Therefore, the argument goes, media capture prevents the citizens from accessing the information required to keep the politicians accountable, hindering the essence of democracy.

This paper proposes an alternative view about media capture. It is based on the idea that politician's decisions of whether to capture media and, conditional on capturing, how to send news are strategic decisions. This is because the politician captures media in order to *persuade* the citizens to support her, which is a strategic process (Kamenica and Gentzkow, 2011; Rayo and Segal, 2010). According to this view, which we will denote the *persuasion view*, it is not always the positive news which will be produced when the media is captured. Instead, the politician would sometimes act strategically and send negative signals about herself in order to be more persuasive. Consequently, media can still be informative even when it is captured. The informativeness of captured media depends on how easy it is for the politician to persuade the citizens, which in turn depends on the perceptions of the citizens about the politician. This naturally bears the question of how a change in citizens' perceptions (an increase in the popularity of politician or a polarization of opinions among the citizens) affects the occurrence of positive and negative news.

In order to model the persuasion view, we present a model of Bayesian persuasion as in Kamenica and Gentzkow (2011). The politician wants the citizens to take a certain action, which corresponds to supporting a policy proposed by the politician. The benefit of the policy is unknown to the citizens, but the politician can send messages about the benefit through the captured media. Unlike the models of Bayesian persuasion in the literature, we consider citizens who have heterogeneous priors about the benefit of the policy, because they have differing views about the politician. We characterize the equilibrium of this model and identify the conditions for media to always send truthful news (i.e. to be fully informative), or the conditions for the media to send the same type of news regardless of the truth (i.e. to be fully uninformative). We then investigate how the equilibrium depends on the distribution of prior beliefs. Interestingly, the politician is more likely to send negative news about herself when there are more citizens with extreme opinions. The occurrence of negative news is an outcome of strategic decisions: the politician realizes that the abundance of negative news makes the rare positive news more convincing for the extreme opposers.

At the heart of our model is a prominent trade-off faced by the politician. It is the result of a simple observation: even when the politician has full control over the media source, she cannot fully control the citizens, who are free in their decisions to pay attention to the media source. A citizen wants to pay attention to the media source, however, in order to be informed. If the media sends nothing but positive news, it will not be informative at all and the citizens would stop paying attention to the news. In order to convince citizens to listen to the news, then, the politician needs to ensure that the media is sufficiently informative: the media should commit to sending truthful news at least occasionally. On the other hand, the politician wants to send favorable news about herself whenever possible in order to persuade citizens. Our Bayesian persuasion model succinctly captures this trade-off. The politician ends up solving an optimization problem where she balances the objectives of "convincing more people to listen to the media" (i.e. gaining in the *extensive margin*) and "sending a favorable message to those who listen to the media" (i.e. gaining in the *intensive margin*).

After setting up the politician's problem, we begin the analysis by identifying the conditions for the captured media to commit to sending truthful news or sending the same type of news regardless of the state (Proposition 1). We show that if there are increasingly more supporters (represented by an increasing density of prior distributions), then the media always sends the same type of news, and no information is revealed. This is because the politician enjoys enough support and she wants to ride the priors of citizens, which can be achieved by providing them as little information as possible and having them not listen to the news at all. On the contrary, if there are increasingly more opposers (represented by a decreasing density), captured media will always send truthful news: there will be full information revelation. The intuition for this result is that, if there are more opposers in the society, the politician needs to persuade the citizens rather than confusing them. It turns out the most efficient way to persuade the citizens is to be truthful towards the citizens and having everyone listen to the media. Such a full revelation result typically does not appear in the standard Bayesian persuasion results with one receiver: it occurs purely because there are multiple receivers with heterogeneous priors.

In our next set of results (Propositions 2 and 3), we provide characterizations of optimal media policies for single-peaked and single-dipped distributions of priors beliefs. A single-peaked distribution of prior beliefs corresponds to a society with many moderate citizens, whereas a single-dipped distribution suggests the existence of more extremists than moderates. An implication of the characterization results is that, in a society where citizens tend to hold extreme beliefs rather than moderate ones, one is more likely to encounter negative news rather than positive ones. The intuition for this result is that, when there are more people with extreme prior beliefs in the society, the politician faces a tough problem: she needs to convince the extreme opposers to listen to the news, which requires committing to a certain level of truthful revelation. But such a strategy has a risk of alienating the extreme supporters: they may listen to the media and stop supporting the politician if the news are negative. It turns out that the optimal strategy for the politician in this case is to allow a lot of negative news to be produced, so that the occurrence of positive news is an event rare enough to convince the extreme opposers to listen to the media. On the other hand, the negative news are abundant enough so that they do not sway the opinions of extreme supporters. This is the first analysis in the literature on Bayesian persuasion under a heterogeneous audience (Kolotilin et al., 2017) about the effect of changes in the distribution of receiver types on the optimal solution.[1]

Our results shed light on interesting and not entirely obvious relationships between media capture and the distribution of beliefs in society. A recent trend in the distribution of beliefs in modern democracies, which is a topic of increasing interest in the literature, is the increase in the number of people with extreme opinions (Gentzkow, 2016). It is often documented that this phenomenon and negative news are jointly observed. Our finding suggests a surprising channel of causality in this relationship: it may be the politician herself, the party which is the target of negative news, who prefers the occurrence of negative news. Correspondingly, this finding suggests that the occurrence of media sources which send negative news about the politician is not necessarily indicative of media freedom.

Our next set of results are a group of comparative statics regarding the informative content of news. Proposition 4 demonstrates that it is relatively more popular politicians (indicated by the distribution of prior beliefs with more mass towards the right) who suppresses the informative content of the news. The intuition is related to the first set of results we discuss above: an unpopular politician wants to make sure that citizens listen to the media, so she would would allow for information revelation. On the other hand, a sufficiently popular politician suppresses the bad news, to the extent that it would make the media source uninformative, to ensure that citizens act based on their priors. This is a prediction that is not offered by the tyranny view of media capture. It is also suggestive of why some leaders have been involved in capturing and suppressing the media only after they gained enough popularity. Proposition 5 identifies conditions on when polarization of opinions in a society (in the sense of replacing some moderate citizens with extreme opponents or supporters) leads to less informative content of media. It demonstrates that polarization leads to less informative news only if polarization generates more opposers than supporters. The intuition is as follows: when most of the moderate citizens turn into extreme opponents, the politician has fewer marginal citizens to convince to listen to the media. On the other hand, there is roughly the same number of supporters who already listen to the media. To avoid antagonizing the supporters, therefore, the politician chooses a less informative media source, and she is more lenient on losing the marginal citizens. If polarization generates more supporters than opposers, this result is reversed: there are already many extreme supporters, so the politician can afford to cater to the marginal citizens by increasing the informative content of the news.

Our final result (Proposition 6) demonstrates that if the media becomes less accessible to the citizens (due to cognitive or monetary costs associated with paying attention to the news), it becomes more informative. Intuitively, this is because a higher cost of listening to media implies that citizens with extreme opinions never follow the news. This leads to

[1] Kolotilin (2015) contains a comparative statics analysis on the distribution of types in a Bayesian persuasion model with heterogeneous audience, but its results are about the monotonicity of welfare, not the properties of solution.

the politician being less concerned about losing the support of the extreme supporters, and she has incentives to make the media more informative to convince the moderate citizens to listen to the media. Interestingly, this finding suggests that accessibility of the media and its informativeness may be negatively associated. It is a prediction that models of media capture without any strategic considerations does not make, and, as it is the case with the other testable predictions of this model, opens up potential avenues for empirical research.

## 1.1 Relation to the Literature

Our model is a Bayesian persuasion model with heterogeneous priors. The two building blocks of the model (Bayesian persuasion, and heterogeneous priors) has been discussed in different literatures extensively, but ours is among the first ones which put these two pieces together. Prominent examples of the Bayesian persuasion literature rely on the common prior assumption, whereas the literature on heterogeneous priors has not been involved with the analysis of persuasion.

This paper is most closely connected to the Bayesian persuasion literature, where the sender is trying to persuade the receiver to take the action preferred by the sender (Rayo and Segal, 2010; Kamenica and Gentzkow, 2011). Such baseline models consider just one sender and one receiver, whereas we consider an audience consisting of heterogeneous receivers. Another paper which considers a case with heterogeneous audience is Kolotilin et al. (2017). In that paper, the audience is heterogeneous because each receiver has private information about her (payoff-relevant) type. Whereas our model can be reformulated such that the prior beliefs can be interpreted as private types, our analysis is concerned with the properties of optimal solution and comparative statics with respect to the distribution of types. In contrast, Kolotilin et al. (2017)'s main contribution is demonstrating the equivalence of public and private persuasion mechanisms. The idea between such an equivalence also provides a foundation for our result on sufficiency of only two messages in our model (Lemma 1).

Kolotilin (2015) is another model which allows private persuasion. Similar to our methodology, Kolotilin (2015) also investigates the potential effects of changes in the distribution of receiver types; but, it is exclusively focused on the effects on measures such as the ex ante welfare of the sender and receiver, rather than inspecting their effects on optimal disclosure strategies. Kolotilin (2018) also considers a model where the receiver has a private type. Differently from our model, the types and state of the world are jointly distributed – in other words, the type of receiver depends on the state of the world in a stochastic manner. Consequently, the sender would find it beneficial to condition the signal distribution on the state in order to elicit the private type.

Our model is also reminiscent of the Gehlbach and Sonin (2014) model on government control of the media, where the authors assume that the politician has commitment power as well. The main difference between our setup and theirs is: we assume *heterogenous priors* on the receiver side, so that a given signal distribution may have different effects in different people's beliefs (and consequently, actions). This is not a very stringent approximation of reality, especially when one considers that typically people have diverse opinions about the quality of the politician or the potential outcomes of the actions taken by the politician. A sort of heterogeneity on the receiver side (which implies that only a certain subset of the population watches the news) is modeled through heterogeneous costs of listening to media in (Gehlbach and Sonin, 2014). Having heterogeneous priors instead of heterogeneous costs allows us to identify who watches the news and who does not, which is useful for our purposes. Differently from Gehlbach and Sonin (2014), we are interested in: how the distribution of priors (i.e. the initial popularity of politician) influences the informativeness of media, and how the informativeness of media influences who watches the media. None of these questions can be answered without having a model with heterogeneous priors.

Finally, the canonical model of media capture, Besley and Prat (2006), also investigates a politician's incentives to capture the media, but it does not model the media capture as an information revelation problem.

# 2 The Model

In this section, we present the problem faced by a politician who has total control over a media source's editorial policy and who wants to persuade the receivers to support her by sending positive news. We demonstrate that the persuasion setup can be reduced to a simple optimization problem where the politician trades off "convincing more people to listen to the media" and "sending a favorable message to those who listen to the media".

## 2.1 Notation and Assumptions

We model the problem faced by a politician as a Bayesian persuasion problem towards an audience with heterogeneous priors. The politician wants the citizens to take an action which is favorable towards herself. This corresponds to supporting a policy proposed by the politician. Obtaining support requires persuading the citizens that the state is "good", i.e. the policy is beneficial for the citizens. As standard in the Bayesian persuasion literature, we assume that the sender can commit to any distribution of messages conditional on the state. Unlike the canonical models of Bayesian persuasion, we assume that there are multiple receivers with heterogeneous priors. That is, the receivers disagree about the ex ante likelihood of the good state; hence, even though they observe the same message, their interpretations differ.

The model has two type of agents: the politician (denoted by $p$), and a continuum of citizens, denoted by $I$. We'll denote a generic citizen as $i \in I$. The total measure of citizens is normalized to one.

There is an underlying state of the world, denoted by $\theta \in \{0,1\}$. Here, $\theta = 1$ corresponds to the "good" state where the policy is beneficial for the citizens. Conversely, $\theta = 0$ is the "bad" state where the policy is not beneficial. For the purposes of this model, what matters is that if the state is $\theta = 0$, the citizens will keep the status quo, and if the state is $\theta = 1$, they approve the policy. In other words, the politician and the citizens agree on state $\theta = 1$ and disagree on state $\theta = 0$.

To express this notion more formally, assume that the citizen $i \in I$ takes an action $a_i \in [0,1]$. Here, $a_i$ is the "support" that citizen $i$ provides to the policy. Her utility is:

$$u_i(a_i, \theta) = a_i(\theta - c)$$

where $c \in (0,1)$ is the cost of implementing the policy relative to the status quo, and is common knowledge. As a result, parameter $c$ also corresponds to the posterior belief that a citizen needs to have to support the policy: it is a measure of how powerful the status quo is.

The politician cares about maximizing the support, so her utility is:

$$u_p(\{a_i\}_{i \in I}) = \int_{i \in I} a_i di$$

This would be easy to analyze if the state is observed by all agents, but the state is not observed by the citizens. Only the politician observes the state, so the citizens need to act based on their beliefs about $\theta$. The politician is able to send informative messages through the media. Assume that the politician can send a message $s \in \{g, b\}$ in each state, where $s = g$ corresponds to a "good" message which suggests that $\theta = 1$, and $s = b$ is the "bad" message which suggests that $\theta = 0$.[2] We assume that the politician can *commit* to generating any distribution of signals conditional on each state, before the state is realized. This implies that the politician's strategy can be expressed as a pair:

$$\sigma = (\sigma_0, \sigma_1) \in [0,1]^2$$

where $\sigma_0 =: Pr(s = g|\theta = 0)$ is the probability of sending good message in the bad state, and $\sigma_1 =: Pr(s = g|\theta = 1)$ is the probability of sending a good message in the good state. The politician's strategy is perfectly observable by each citizen.

Observing the politician's strategy/editorial policy, each citizen $i \in I$ decides whether to watch the news or not. Watching the news costs $\kappa > 0$ to each citizen. For the baseline

[2] As in every communication model, for each equilibrium, there is a corresponding "mirror-image" one which we don't consider.

model, we will assume that $\kappa$ is an infinitesimally positive amount, which is the cognitive cost of watching the news. The implicit assumption here is that there is no monetary cost of following the news (consider the media source as public television), and processing the news does not impose a significant cost on the citizens. In Section 6, we provide a generalization to the case where listening to media costs any positive amount $\kappa > 0$.

Watching the news is helpful for the citizens to the extent that it potentially changes the action the citizen will take. This, clearly, depends on the distribution of posteriors of agent $i$, which in turn depends on her prior. The citizens have *heterogenous priors* about the state of the world. That is, the ex ante probability of good state, $Pr\{\theta = 1\}$, differs by citizen and is indexed by $i$. For each $i \in I$, let $p_i := Pr_i\{\theta = 1\} \in [0, 1]$ denote the prior (subjective) probability that citizen $i$ assigns to the good state. Assume that the priors are drawn independently from a distribution $F(.)$ with support $[0, 1]$, that is:

$$p_i \sim F(.) \; \forall i \in I$$

Along with the assumption that the total measure of citizens is normalized to one, this means that the measure of citizens with prior less than or equal to $p$ is $F(p)$. In order to avoid measurability issues, we assume that the pdf of $F(.)$, $f(.)$, is continuously differentiable. We will focus on single-peaked or single-dipped distributions throughout the analysis. Such a focus is appropriate for the types of questions we are asking (such as the effects of polarization), and is relevant for the analysis of political economy in many cases.

The politician's prior is:

$$p^* = 1 - c$$

which is inversely related to the cost of policy: if a policy is more costly (or if the status quo is stronger), the politician is less optimistic about its benefit.[3]

**Timing** The timing of the game is as follows.

1. The prior of each citizen is drawn, and each citizen $i \in I$ observes her prior. The distribution $F(.)$ is common knowledge.
2. The politician commits to a strategy $(\sigma_0, \sigma_1)$, which is observed by each citizen.
3. Each citizen $i \in I$ decides whether to watch news or not.
4. The state is realized, and the media sends the message drawn according to the politician's strategy.
5. Each citizen $i \in I$ updates her prior, and chooses the action $a_i$ based on the posterior.

[3]One may microfound this formulation by assuming that the politician considers the benefit from the policy to be uniformly distributed between 0 and 1.

6. Payoffs are realized.

The solution concept we will adopt is Perfect Bayesian Equilibrium.

## 2.2 Discussion of The Model

The baseline model contains four crucial features of the model that are worthy of some discussion on their own. The first one is the ability of the politician to commit to any conditional distribution of messages. The second one is the assumption that the politician can generate only two types of messages (good or bad). Finally, the question of whether more general forms of payoff functions for the citizens and for the politician can be adopted remains. We discuss the justifications behind our modeling choices and the implications of relaxing them.

**Commitment** The justification for the commitment assumption comes from how media capture works in practice. Even when the politician captures the media, she cannot check every single news produced by the state television. Nevertheless, she can have a general control over the media source's editorial policy and the intensity of control is observable by the citizens. Of course, the intensity of control depends on the state of the world (i.e. the information that the state television has access to), hence the signal distribution depends on $\theta$.[4] The commitment to a strategy $(\sigma_0, \sigma_1)$, then, should be interpreted as commitment to an editorial policy which is observable to the citizens.

The technical advantage of this assumption is that it allows us to reduce the decision of politician to a standard constrained optimization problem (Section 2.3). In this sense, it allows us to use the machinery used in the Bayesian persuasion literature (Kamenica and Gentzkow, 2011) which makes the analogous assumption.

**Two Messages** The assumption of having as many messages as the number of possible states is also one adopted by Kamenica and Gentzkow (2011). The authors justify their assumption by deriving a result which closely resembles *revelation principle* (Proposition 1). Their result depends on having only one receiver, and it breaks down in the case of multiple receivers with heterogeneous priors. If more messages are available for the politician, she would be able to induce some action profiles she is not able to induce with two messages only. The following result argues that, in the set of distributions we consider, having two messages does not have impose a consequential restriction.

[4]One alternative modeling strategy is to model this as a *cheap talk* game, which does not allow for commitment on the politician's side. This requires that the signal distributions in each state must be the same ($\sigma_1 = \sigma_0$), thus we would be forgoing at least some of the richness of the model.

**Lemma 1.** *Let $S$ be a set of signals with $|S| \geq 2$, and assume the politician commit to any signal distribution*

$$(\sigma_0, \sigma_1) \in \Delta(S) \times \Delta(S)$$

*If $f(.)$ is single-peaked or single-dipped, the optimal strategy can be implemented using only two signals.*

*Proof.* See Appendix A. The proof is an adaptation of Theorem 1 in Kolotilin et al. (2017), which proves an equivalence result in a Bayesian persuasion model with heterogeneous audience but common priors. □

The assumption of having only two messages can also be justified on the grounds that the citizens are subject to some cognitive constraints. One may imagine that it is impossible for a citizen to distill every detail in the news and form a nuanced view of the state of the world. Instead, each citizen reads watches the news and leaves with some takeaway in the form of “the state if good” or “the state is bad”.

**Payoff Functions of Citizens** The current form of payoff functions of citizens, which adopts linear costs, tends to lead to corner solutions. That is, we will generically observe $a_i \in \{0, 1\}$ in the equilibrium. This is a deliberate modeling choice. Here is what goes wrong if an alternative payoff function is adopted. Suppose that we change the utility function of citizen $i$ to:

$$u_i(a_i, \theta) = -(a_i - \theta)^2$$

The quadratic loss function implies that the best action of the agent is no longer the “corner” action; rather, it equals to the posterior belief given the observed message. This has two implications.

1. Because the agents will necessarily act based on the posterior, everyone will watch the news even if they are a little bit informative.[5] Consequently, the main trade-off of politician will be lost.
2. Because each citizen is Bayesian, her beliefs are a martingale with respect to her own prior. Therefore, when her action equals her posterior belief, the expected action for each citizen equals to her prior. But then, a politician with linear utility will be indifferent among any posterior distribution, so the politician will not care about the informativeness of media at all.[6] We conclude that some level of “coarseness” of the action is necessary for the persuasion setup to be nontrivial. As long as such coarseness is maintained, the insights generated in specific case with only two actions apply.

[5]This would change with $\kappa >> 0$.

[6]This would change if the politician cares about the revenue obtained by media, as in Gehlbach and Sonin (2014).

**Payoff Function of Politician** The baseline model assumes linear utility on the politician's side. That is, the politician cares about the sum of all $a_i$'s, rather than a more sophisticated functional form. This is assumption is crucial for the model to be operational. Suppose that we change the utility function of the politician to:

$$u_p(\{a_i\}_{i \in I}) = -\int_{i \in I} (1 - a_i)^2 di$$

In this case, the quadratic loss function would make the politician "risk-averse" with respect to each citizen's action. Consequently, the politician would enjoy as little variation as possible in the action of each citizen from an ex ante perspective. She would therefore prefer to make the media as uninformative as possible, in order to keep the posteriors close to prior. In equilibrium, one would have an uninformative media which nobody follows.[7]

## 2.3 The Optimization Problem

We now describe how the politician's choice of editorial policy can be expressed as a constrained optimization problem, which simplifies the analysis considerably. The simplification works in two steps: we first identify which citizens would listen to the media given an editorial policy, and then present the problem of finding the optimal policy of the politician who takes the citizens' decisions into account. The optimization problem we obtain succinctly captures the main trade-off of media capture: the politician needs to make the media informative enough so that it convinces the citizens to listen to the news, but still distort the information such that good messages are sent frequently.

**The Decision of Listening Given $(\sigma_0, \sigma_1)$:** We begin by pinning down a strategy of the politician $(\sigma_0, \sigma_1)$ first; we will characterize program which picks the optimal pair $(\sigma_0, \sigma_1)$ later.

Consider a citizen $i \in I$ and her posterior belief about the probability of good state following a message $s$, $Pr_i(\theta = 1|s)$. The utility function of the citizen suggests the following:[8]

$$a_i = \begin{cases} 1 & if \quad Pr_i(\theta = 1|s) \geq c \\ 0 & otherwise. \end{cases}$$

[7] This would probably change with coarser actions, but adopting a convex loss function instead of a linear one would not provide any insights not given by our baseline model.

[8] The agent is indifferent between any actions when her posterior equals $c$. Consequently, there is an indeterminacy in this case. We stay away from this indeterminacy by simply that the citizen takes politician's favorite action when indifferent, as in Kamenica and Gentzkow (2011). This assumption can be justified on the grounds that the politician can always invoke a posterior $c + \epsilon$ for any $\epsilon > 0$, thus having the agent strictly prefer $a_i = 1$.

We now consider how the posteriors are calculated. If a citizen $i$ does not watch the news, she will have her posterior equal to $p_i$ following any message. If citizen $i$ watches the news, her posterior will depend on the message $s$ she observes. In this case:

$$
\begin{aligned}
Pr_i\{\theta = 1|s = g\} &= \frac{p_i\sigma_1}{p_i\sigma_1 + (1-p_i)\sigma_0} \\
Pr_i\{\theta = 1|s = b\} &= \frac{p_i(1-\sigma_1)}{p_i(1-\sigma_1) + (1-p_i)(1-\sigma_0)}
\end{aligned}
$$

Note that, given $(\sigma_0, \sigma_1)$, both values are increasing in $p_i$. Moreover, assuming $\sigma_1 \geq \sigma_0$,[9] we have:

$$Pr_i\{\theta = 1|s = g\} \geq p_i \geq Pr_i\{\theta = 1|s = b\}$$

That is, good news update the prior upwards, and bad news update it downwards, as expected.

Now we characterize when citizen $i$ decides to watch the news. Given the infinitesimal cost $\kappa > 0$ of watching the news, a citizen $i$ watches the news if and only if it would change the citizen's action with some probability. This is possible if and only if $i$ takes different actions following each message. That is, $i$ watches news if and only if:

$$Pr_i\{\theta = 1|s = g\} \geq c \geq Pr_i\{\theta = 1|s = b\}$$

Substituting the above expressions, this condition becomes:

$$\frac{p_i\sigma_1}{p_i\sigma_1 + (1-p_i)\sigma_0} \geq c \geq \frac{p_i(1-\sigma_1)}{p_i(1-\sigma_1) + (1-p_i)(1-\sigma_0)}$$

Rearranging, we end up with the following Lemma.

**Lemma 2.** *Given $(\sigma_0, \sigma_1)$ where $\sigma_1 \geq \sigma_0$, a citizen $i \in I$ with prior $p_i$ watches the news if and only if $p_i \in [\underline{p}, \bar{p}]$, where:*

$$
\begin{aligned}
\underline{p} &= \frac{c\sigma_0}{c\sigma_0 + (1-c)\sigma_1} \\
\bar{p} &= \frac{c(1-\sigma_0)}{c(1-\sigma_0) + (1-c)(1-\sigma_1)}
\end{aligned}
$$

Those with $p_i < \underline{p}$ do not watch the news because they will set $a_i = 0$ even if they would observe the good news: they simply don't believe in the politician because their initial beliefs are too pessimistic. Similarly, those with $p_i > \bar{p}$ do not watch the news because they will set $a_i = 1$ even if they would receive bad news: they support the politician regardless of the

[9] Without loss of generality, since in the opposite case, the politician can generate the same distribution of posteriors by adopting an alternative strategy where $\sigma_1 \geq \sigma_0$.

information obtained from the news.[10]

Given this discussion, the measure of citizens who watch news is:

$$F(\bar{p}) - F(\underline{p}) = F\left(\frac{c(1-\sigma_0)}{c(1-\sigma_0)+(1-c)(1-\sigma_1)}\right) - F\left(c\frac{\sigma_0}{c\sigma_0+(1-c)\sigma_1}\right)$$

These citizens act based on the information obtained from the news: they set $a_i = 1$ if and only if the message is good. In addition, a measure $1 - F(\bar{p}) = 1 - F\left(\frac{c(1-\sigma_0)}{c(1-\sigma_0)+(1-c)(1-\sigma_1)}\right)$ of citizens always set $a_i = 1$ without watching the news.

**Setting** $(\sigma_0, \sigma_1)$**:** Now we consider the politician who picks a strategy to commit to. If the politician's prior about the state of the world is $p^* = 1 - c \in [0, 1]$, by committing to $(\sigma_0, \sigma_1)$, the politician sends a good message with probability $p^*\sigma_1 + (1 - p^*)\sigma_0$. All in all, the expected support for policy is:

$$\left(F(\bar{p}) - F(\underline{p})\right)(p^*\sigma_1 + (1-p^*)\sigma_0) + (1 - F(\bar{p}))$$

A politician with prior $p^*$ therefore solves the following optimization problem when choosing the editorial policy $(\sigma_0, \sigma_1)$:

$$\max_{(\sigma_0,\sigma_1)\in[0,1]^2,\sigma_1\geq\sigma_0} \left(F\left(\frac{c(1-\sigma_0)}{c(1-\sigma_0)+(1-c)(1-\sigma_1)}\right) - F\left(\frac{c\sigma_0}{c\sigma_0+(1-c)\sigma_1}\right)\right)(p^*\sigma_1 + (1-p^*)\sigma_0)$$
$$+\left(1 - F\left(\frac{c(1-\sigma_0)}{c(1-\sigma_0)+(1-c)(1-\sigma_1)}\right)\right)$$

The solution clearly depends on the value of $p^* = 1 - c$ and $F(.)$. Our approach would be fixing $c$ and observing how the solution changes with $F(.)$. On the practical level, this corresponds to investigating how the informativeness of media changes when people's opinions about a politician change.

The broad insight about the politician's problem is that: in the optimal solution, she finds the balance in an *intensive* versus *extensive* margin trade-off. In the absence of any strategic considerations, the politician would send only positive news, i.e. she would set $\sigma_0 = \sigma_1 = 1$. When citizens are strategic in their decisions, however, such a strategy would result in no one listening to the media, i.e. it would lead to $\underline{p} = \bar{p} = c$. In this case, the politician would like the media to be more informative, which would be reflected in a decrease in $\sigma_0$. By decreasing $\sigma_0$, the politician would ensure more people listening to the media (in the form of a decrease in $\underline{p}$), which correspond to gaining people in the *extensive margin*. On

[10] As a sanity check, note that when $\sigma_1 = \sigma_0$ (i.e. when media is totally uninformative), we have: $\underline{p} = \bar{p} = c$ – no one (except possibly for a measure zero of agents) watches the news. Conversely, when $\sigma_1 = 1$ and $\sigma_0 = 0$ (i.e. when media is fully informative), we have: $\underline{p} = 0$ and $\bar{p} = 1$ – everyone watches the news.

the other hand, the occurrence of good news would decrease, and each citizen who listens would be supporting the policy with a lower likelihood, i.e. the politician would lose people in the *intensive margin*. The optimal solution balances these two effects. The figure below illustrates this idea. For illustration purposes, it fixes $\sigma_1 = 1$ (which leads to $\overline{p} = 1$) and considers the optimal choice of $\sigma_0 \in [0, 1]$, which maps into the optimal choice of $\underline{p}$.

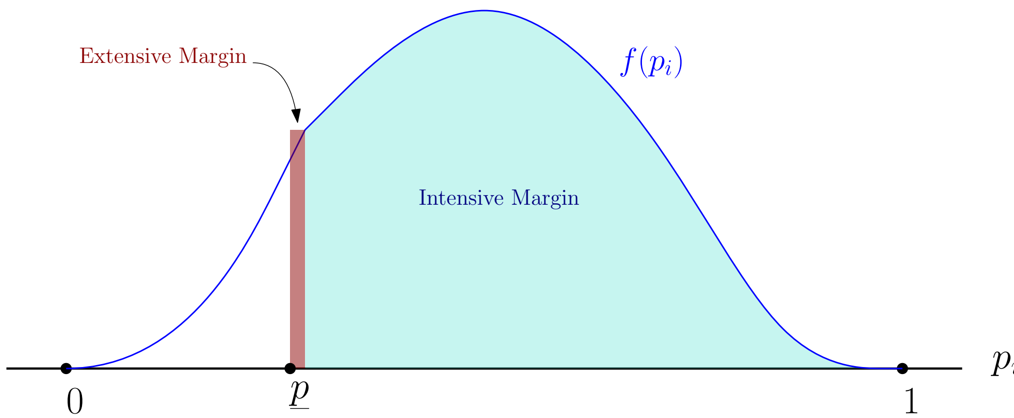


Figure 1: The intensive versus extensive margin trade-off.

# 3 The Solution

In this section, we sketch the universal features of the solution to the politician's problem. In particular, we demonstrate that for the class of prior distributions we consider, the solution lies in the extreme: it either reveals one of the states perfectly (i.e. one of the messages is fully informative) or it contains no information at all. To set up this result, we first provide a geometric interpretation of the politicians constrained optimization problem and offer an interpretation for the boundary conditions. We then demonstrate that if the prior distribution is single-peaked or single-dipped, the solution must lie in one of these boundaries: we cannot have an interior solution.

## 3.1 Simplifying the Problem

Let us begin by providing a geometric interpretation of the politicians problem. The constrained optimization problem can be represented as a problem on a two-dimensional plane whose constraints form a triangle. Each edge of the triangle represents an extreme information structure, and these extremes will correspond to potential solutions of the problem.

Substituting in $p^* = 1 - c$ into the objective function of politician and rearranging,[11] one can write the politician's problem as:

[11] The step-by-step simplification is provided in Appendix B.

$$\min_{(\sigma_0,\sigma_1)\in\Delta} \Pi(F,(\sigma_0,\sigma_1)) \tag{1}$$

where

$$\Pi(F,(\sigma_0,\sigma_1)) := F\left(\frac{c(1-\sigma_0)}{c(1-\sigma_0)+(1-c)(1-\sigma_1)}\right)(c(1-\sigma_0)+(1-c)(1-\sigma_1)) + F\left(\frac{c\sigma_0}{c\sigma_0+(1-c)\sigma_1}\right)(c\sigma_0+(1-c)\sigma_1)$$

and

$$\Delta := \{(x,y) : x \geq 0, y \geq 0, y \geq x\}$$

Note that $\Delta$, the set of feasible $(\sigma_0,\sigma_1)$ pairs, is geometrically simply a triangle. The interior of this set corresponds to $(\sigma_0,\sigma_1)$ pairs such that $0 < \sigma_0 < \sigma_1 < 1$. Any such pair maps to a range of priors where people listen to media $[\underline{p},\bar{p}]$ with $0 < \underline{p} < c < \bar{p} < 1$. In addition to the interior region, there are three boundaries of this set:

- The first boundary is the line where $\sigma_0 = \sigma_1$. In this boundary, any message sent by the media is ***fully uninformative***, i.e. posterior following any message equals the prior for each citizen who listens to the media. In this boundary, we have $\underline{p} = \bar{p} = c$; consequently, the measure of citizens who listen to the media equals zero.
- The second boundary is the line where $\sigma_0 = 0$. In this boundary, ***the "good" message is fully informative***, in the sense that $Pr_i(\theta = 1|s = g) = 1$ for each $i \in I$ who listens to the media. In this boundary, we have $\underline{p} = 0$, i.e. even the most pessimistic citizen listens to the media.
- The third boundary is the line where $\sigma_1 = 1$. In this boundary, ***the "bad" message is fully informative***, in the sense that $Pr_i(\theta = 1|s = b) = 0$ for each $i \in I$ who listens to the media. In this boundary, we have $\bar{p} = 1$, i.e. even the most optimistic citizen listens to the media.

The figure provided below illustrates $\Delta$ and summarizes this discussion.

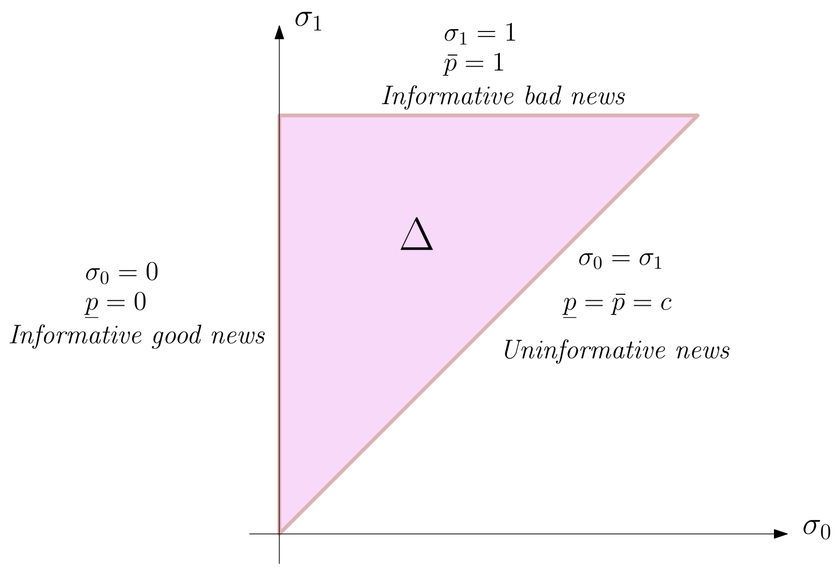


Figure 2: The set of feasible $(\sigma_0, \sigma_1)$ pairs.

## 3.2 Discarding Interior Solutions

Having Equation 1 in hand, we now proceed to analyze the properties of the solution. The following result demonstrates that if the distribution of priors is single-peaked or single-dipped, the solution cannot be interior and we need to look for the boundaries of $\Delta$ to find the solution.

**Lemma 3.** *Suppose $f(.)$ is single-peaked or single-dipped. Then, the politician's problem cannot have an interior solution. That is, the optimal solution has one of the following three forms.*

- $\sigma_0 = \sigma_1$, *where the messages are fully uninformative,*
- $\sigma_0 = 0$, *where the "good" message is fully informative,*
- $\sigma_1 = 1$, *where the "bad" message is fully informative.*

*Proof.* See Appendix C. □

Lemma 3 is extremely useful because it reduces the optimization problem into a one-dimensional problem: all one needs to do is to check the boundaries of $\Delta$, discussed above. In Appendix D, we discuss how this problem can be reduced to a tractable one-dimensional optimization problem through appropriate changes of variables.

# 4 Characterization Results

With the simplified optimization problem in hand, it is now the time to characterize the solutions. In this section, we identify the general features of the optimal policy given the prior distribution. Proposition 1 characterizes the solution for the cases of monotonically increasing or monotonically decreasing densities: in the former case the optimal policy is to reveal nothing, and in the latter case the optimal policy is to reveal the state perfectly. Propositions 2 and 3 are weaker characterizations for the single-peaked and single-dipped distributions, respectively. In the case of single-peaked distributions, the“bad” message is fully informative; in the single-dipped case, the “good” message is.

**Proposition 1.** *If $f(.)$ is monotonically increasing in $[0, 1]$, then the optimal strategy has $\sigma_0^* = \sigma_1^*$, i.e. the media is fully uninformative. If $f(.)$ is monotonically decreasing in $[0, 1]$, then the optimal strategy has $\sigma_0^* = 0$ and $\sigma_1^* = 1$, i.e. the media is fully informative.*

*Proof.* See Appendix E. □

Proposition 1 offers sharp sufficient conditions for the shape of the solution. It suggests that if beliefs are skewed in favor of the politician, the politician makes the media fully uninformative, so that nobody follows the media. If, on the other hand, beliefs are skewed against the politician, the politician makes the media fully informative, so that everybody follows the media and they always learn the truth.

In order to see the intuition behind this results, one can revisit to the Kamenica and Gentzkow (2011)'s Bayesian persuasion setup. The following is a very simple but substantial insight offered by Kamenica and Gentzkow (2011): the extent of persuasion depends on the original belief differences between the sender and the receiver. If the receiver agrees with sender ex ante, no persuasion will be observed in equilibrium: indeed, the sender would like to reveal as little information as possible and “ride the prior” of the receiver. On the other hand, if the receiver and sender disagree ex ante, there will be some persuasion, and hence some information revelation, in equilibrium. As the receiver becomes more pessimistic about the state ex ante, the sender will choose a more informative revelation strategy. Looking at Proposition 1 through this lens makes the intuition clear. When the citizens are ex ante more in favor of the politician, there is no persuasion and information revelation – instead, what we will observe is people acting based on their priors. When the citizens are ex ante less in favor of the politician, the politician makes her best to convince people and reveal as much information as possible.

The following is another result which has a similar flavor as Proposition 1. The conditions it employs are much weaker than the monotonicity conditions required in Proposition 1, hence the predictions are also weaker. Nevertheless, a tight characterization is still possible, and the general predictions are in line with those made in Proposition 1.

**Proposition 2.** *Suppose $f(.)$ is single-peaked.*

- *If $F(c) > c$, then there will be* ***some*** *information revelation. That is, the messages cannot bu fully uninformative. The optimal solution has $\sigma_1^* = 1$ (i.e. the "bad" message is fully informative) and $\sigma_0^* \in [0, 1)$.*
- *If $F(c) < c$, then there will* ***never*** *be full information revelation. The optimal solution has $\sigma_1^* = 1$ (i.e. the "bad" message is fully informative) and $\sigma_0^* \in [\frac{c-F(c)}{c}, 1]$.*

*Proof.* See Appendix F. □

Note that the condition $F(c) > c$ can equivalently be expressed as "less than $c$ fraction of citizens support the policy ex ante". Therefore, this should be read as a condition on the level of ex ante support the policy enjoys.

Note that any equilibrium of a single-peaked distribution is that has $\sigma_1^* = 1$. That is, as long as $\sigma_0^* \in [0, 1)$, one has $\bar{p} = 1$ in equilibrium: even the most optimistic citizens will follow the news. The intuition between this result is that: with a single-peaked distribution, there are more *moderate* citizens with priors close to $c$. These citizens are the ones who are more inclined to listen to the news, and they are more sensitive to the news. As a result, the *intensive margin* becomes the more important one, and the setup becomes a generalization of Kamenica and Gentzkow (2011). The politician can set $\sigma_1 = 1$ and choose a sufficiently high $\sigma_0 \in [0, 1]$ to ensure that positive news are generated sufficiently frequently (i.e. the gain the intensive margin is enough) and the losses in the extensive margin equals the gains in the intensive margin (i.e $\underline{p}$ is sufficiently high). The illustration in Figure 1, it turns out, represents the optimal choice of the politician. In the optimal policy, the gains from having more people listen to the media (the mass of citizens in the red slice) equals the expected loss of people who are informed about the media (the likelihood of citizens in the blue region supporting the policy).

Even though $\sigma_1^* = 1$ is pinned down in equilibrium, pinning down the exact value of $\sigma_0^*$ is more difficult. Nevertheless, it is possible to identify some sufficient conditions. For instance, in the first case where the policy does not enjoy ex ante support, the following condition is sufficient to ensure that $\sigma_0^* = 0$:

$$F(x) \geq x \quad \forall x \in [0, c]$$

This is equivalent to $F(.)$ being first-order stochastically dominated by the uniform distribution in $[0, c]$. Intuitively, this means that $F(.)$ has "sufficient mass on the left", i.e. there are enough people with unfavorable ex ante beliefs. In general, though, there are examples where $\sigma_0^* > 0$. An obvious one is the toy model discussed in the introduction of Kamenica and Gentzkow (2011), which is indeed a special case of the model considered here.

In the second case where the policy enjoys some ex ante support, it is possible to set a lower bound for $\sigma_0^*$. As one can see in the statement of Proposition 2, $\frac{c-F(c)}{c}$ is such a lower bound. (This lower bound is not tight and in many cases, one may have $\sigma_0^* = \sigma_1^* = 1$.) Realize that as $F(c) \to 0$, this lower bound converges to one. That is, as the mass of citizens who ex ante oppose the policy shrinks, the media is likely to get less informative – in the limit, when $F(c) = 0$, no information is revealed and everybody supports the politician.

Below is a "mirror image" of Proposition 2, which investigates the case of single-dipped distributions.

**Proposition 3.** *Suppose $f(.)$ is single-dipped.*

- *If $F(c) > c$ (i.e. if the policy does not enjoy ex ante support), then there will be* ***some*** *information revelation. That is, the messages cannot bu fully uninformative. The optimal solution has $\sigma_0^* = 0$ (i.e. the "good" message is fully informative) and $\sigma_1^* \in [\frac{F(c)-c}{F(c)-F(c)c}, 1]$.*
- *If $F(c) < c$ (i.e. if the policy enjoys ex ante support), then there will* ***never*** *be full information revelation. The optimal solution has $\sigma_0^* = 0$ (i.e. the "good" message is fully informative) and $\sigma_1^* \in [0, 1)$.*

*Proof.* See Appendix G. □

Most of this result has an obvious parallel to Proposition 2, so all the comments we made earlier on Proposition 2 can be adapted to here. For instance, the first part of Propositon 3 provides a lower bound for $\sigma_1^*$, and this lower bound converges to 1 as $F(c) \to 1$. That is, as the mass of citizens who ex ante oppose the policy grows, the media is more likely to get more informative.

Any equilibrium of a single-dipped distribution has $\sigma_0^* = 0$ and, as long as $\sigma_1^* \in (0, 1]$, one has $\underline{p} = 0$ in equilibrium. Therefore, even the most pessimistic citizens will follow the news. In contrast to the case with a single-peaked distribution, there are not many moderate citizens; instead, now there is an overwhelming measure of *extremist* citizens with priors close to 0 or 1. These citizens are, in general, not inclined to listen to the news unless they are extremely informative in the opposite direction as their priors, e.g. an extremist opposer (with prior $p_i \approx 0$) would listen to the news only if the good news are informative. To convince these citizens to listen to the news, the politician offers informative good news with $\sigma_0^* = 0$. She chooses a sufficiently high $\sigma_1^* \in [0, 1]$ to ensure that positive news are generated sufficiently frequently, yet $\bar{p}$ is sufficiently low so that extremist supporters still act based on their priors. The figure below illustrates the optimal policy in this case. The citizens whose priors are larger than $\bar{p}$ act based on their priors (i.e. they set $a_i = 1$) and the citizens whose priors are smaller than $\bar{p}$ follow the news (i.e. they set $a_i = 1$ only if $s = g$).

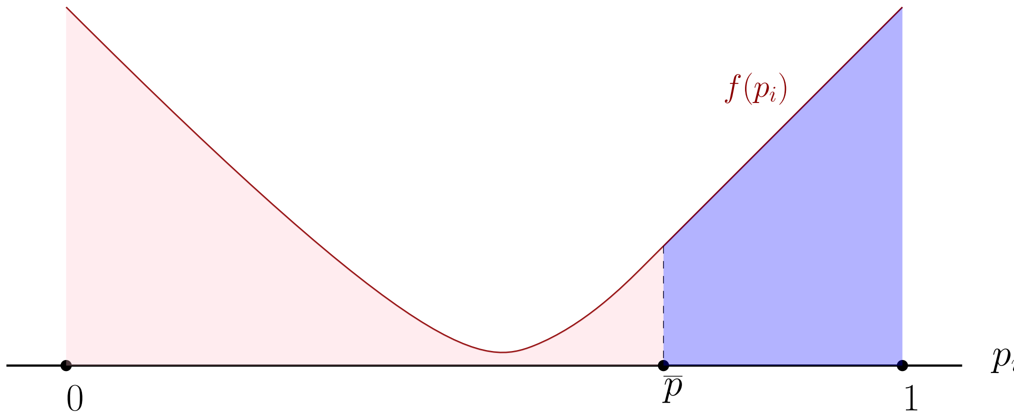


Figure 3: Optimal policy in a single-dipped distribution.

# 5 Comparative Statics

Having characterized the optimal solution for different distributions of priors, we now conduct comparative statics exercises with respect to the distribution of the priors of the citizens.

## 5.1 Effect of Popularity

The first comparative statics exercise we will conduct is regarding the popularity of the politician. We model an increase in a politician's popularity as a shift in the citizen's priors over the right in the MLRP sense. The following result characterizes the changes in the informativeness of the media.

**Proposition 4.** *Consider two distributions $f(.)$ and $\tilde{f}(.)$ satisfying the strict monotone likelihood ratio property, i.e., such that the ratio $f(x)/\tilde{f}(x)$ is strictly increasing in $x$.*

*(a) If $f(.)$ and $\tilde{f}(.)$ are both single-peaked, then $\sigma_0^* > \tilde{\sigma}_0^*$.*

*(b) If $f(.)$ and $\tilde{f}(.)$ are both single-dipped, then $\sigma_1^* < \tilde{\sigma}_1^*$.*

*Proof.* See Appendix H. □

In both cases covered in Proposition 4, the media becomes less informative as the citizens become more optimistic about the politician. The intuition follows from the one we provided after Proposition 1: a less popular politician needs to convince the citizens to listen to the news, which requires providing an informative media. On the other hand, a more popular politician can afford to provide a less informative media and *ride the priors* of citizens.

Our interpretation for Proposition 4 is that only sufficiently popular politicians are bothered by an independent media which provides informative news. Indeed, if there is a positive cost of capturing the independent media, only sufficiently popular citizens would be willing to pay this cost. This reasoning suggests that media capture is a concern only when the politician is popular. This is a prediction that the canonical view of media capture, the tyranny view, does not provide. In our model, media capture and popularity of politicians appear as complements rather than substitutes. This dynamic sheds light to an interesting dynamic observed in modern democracies, where the concerns about media capture became more prevalent as the politicians became more popular over time in their respective countries and enjoyed more vocal support. The empirical support behind this prediction warrants further analysis.

## 5.2 Effect of Polarization

One qualitative difference between the optimal policies in the single-peaked and single-dipped distributions is the abundance of negative news. A group of citizens which primarily involve extremists (as represented by a single-dipped prior distribution) encounters negative news more frequently than a group of citizens which primarily involve moderates (as represented by a single-peaked prior distribution). Interestingly, when the group of citizens involve extremists, it is the politician herself who sends the negative news. This stands in contrast with the common narrative about the effects of polarization.

Inspired by this observation, we study the effect of continuously deforming a single-peaked distribution of the priors to more closely resemble a uniform distribution. Practically, this corresponds to removing some moderates from the group of citizens and adding some extremists. We do this by considering a flexible family of distributions parameterized by two parameters:

$$F_{\alpha,\rho}(x) = (\alpha S(x)^{\rho} + (1-\alpha)x^{\rho})^{\frac{1}{\rho}} \qquad \alpha \in [0,1], \rho \in \mathbb{R}.$$

Note that $F_{\alpha,\rho}(0) = 0$, $F_{\alpha,\rho}(1) = 1$, and $F'_{\alpha,\rho}(x) > 0$ for all $\alpha \in [0,1]$, so $F_{\alpha,\rho}(0) = 0$ is a proper cdf.

Given $(\alpha, \rho)$, $F_{\alpha,\rho}(x)$ is a mixture of $S(x)$ and (some transformation of) the uniform distribution. Because $S(x)$ is single-peaked, this corresponds to putting more weight on the tails of $S(x)$. Indeed, given $\rho$, decreasing $\alpha$ corresponds to taking some mass from the middle of $S(x)$ and putting it on the extremes. Below is a figure illustrating the case where $\rho = 1$ and $\alpha \in (0,1)$.

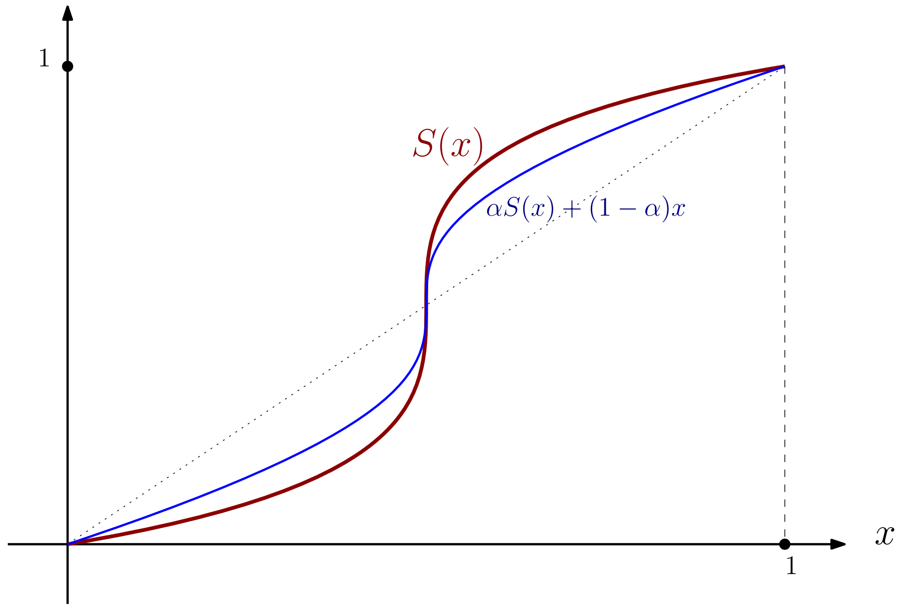


Figure 4: Making the distribution of priors more polarized.

$\rho \in \mathbb{R}$ is a tractable way of parametrizing the weight one puts on either extreme. The easiest way of seeing this is to consider the limits of $\rho$.

- When $\rho = \infty$, $F_{\alpha,\rho}(x) = \max\{S(x), x\}$. Therefore, $F_{\alpha,\rho}(x)$ corresponds to taking some mass from the middle of $S(x)$ and adding it to the left tail. Intuitively, then, $\rho > 1$ is the case where one is taking some of the moderate citizens in the population and turning them into extreme agents (and turning most of them into extreme opponents).
- In contrast, when $\rho = -\infty$, $F_{\alpha,\rho}(x) = \min\{S(x), x\}$. In this case, $F_{\alpha,\rho}(x)$ is taking some mass from the middle of $S(x)$ and adding it to the right tail. Intuitively, then, $\rho < 1$ is the case where one is taking some of the moderate citizens in the population and turning them into extreme agents (and turning most of them into extreme supporters).

Therefore, $F_{\alpha,\rho}(x)$ is a *more polarized* version of $S(x)$, where $\rho > 1$ implies more opponents, and $\rho < 1$ implies more supporters.

The following is the main result of this subsection.

**Proposition 5.** *Assume $S(x)$ the cdf of a single-peaked distribution. Consider the family of distributions defined as $F_{\alpha,\rho}(x) = (\alpha S(x)^\rho + (1-\alpha)x^\rho)^{\frac{1}{\rho}}$. For any $\alpha \in [0,1]$ and $\rho \in \mathbb{R}$, $f_{\alpha,\rho}(.)$ is single-peaked, and thus $\sigma_1^* = 1$. Furthermore,*

*(a) If $\rho = 1$, then $\sigma_0^*$ is independent of $\alpha$.*

*(b) If $\rho > 1$, then $\sigma_0^*$ is decreasing in $\alpha$.*

*(c) If $\rho < 1$, then $\sigma_0^*$ is increasing in $\alpha$.*

In light of the discussion above, the statement "If $\rho > 1$, then $\sigma_0^*$ is decreasing in $\alpha$." should be read as follows: *When moderate citizens turn into extremist opponents, media becomes less informative.* The intuition behind this is closely related to the intensive versus extensive margin trade-off we presented earlier. The transformation turns some moderate citizens into extremist opponents who are *lost causes*: before they were keen to listen to the media, but now it is very costly for the politician to convince them to pay attention to the news. Consequently, there are now less people in the extensive margin, but still the same number of supporters who listen to the media already. The trade-off therefore leans toward the intensive margin: the politician would rather have a slightly less informative media and have a higher support from people who listen. For the $\rho < 1$ case, the opposite reasoning applies: some moderate people become fierce supporters with prior 1, so the politician can afford to cater a little bit towards opponents by offering a slightly more informative media.

The effect of polarization on the informativeness of news occurs through a channel that has not been analyzed before in the literature. There is an extensive discussion on polarization leading to less informative news (Sunstein, 2001). Proposition 5 characterizes under what conditions one would expect to see an association between polarization of opinions and less informative news. Moreover, it argues that if such an association is observed, it may be in the politician's best interest to allow for less information revelation. The empirical assessment of this association and the channel through it occurs warrants further analysis.

# 6 Cost of Listening to Media

In this section, we provide a generalization of the baseline model where the cost of listening to the media can take a large positive value. The general insights from the baseline model go through in this case, but this extension allows us to conduct a comparative statics exercise on the cost of listening to the media.

Suppose that there is a cost $\kappa > 0$ of listening to the media. That is, a citizen has to pay $\kappa > 0$ if she chooses to observe the signal. This may correspond to the cognitive cost of listening to the media or the monetary cost, and $\kappa$ in general parametrizes the accessibility of media. Following the same steps as in the derivation of Lemma 2, one can easily prove the following statement.

**Lemma 4.** *Given $(\sigma_0, \sigma_1)$ where $\sigma_1 \geq \sigma_0$, a citizen $i \in I$ watches the news if and only if $p_i \in [\underline{p}, \bar{p}]$, where:*

$$\underline{p} = \min\{\frac{c\sigma_0 + \kappa}{c\sigma_0 + (1-c)\sigma_1}, c\}$$
$$\bar{p} = \max\{\frac{c(1-\sigma_0) - \kappa}{c(1-\sigma_0) + (1-c)(1-\sigma_1)}, c\}$$

Clearly, if $\kappa \geq c(1-c)$, one would have zero measure of citizens following the news: $\kappa$ is simply too high in this case. Otherwise, an increase in $\kappa$ leads to a *shrink* in the $[\underline{p}, \overline{p}]$ range. As a matter of fact, one can simply repeat the analysis with a transformation of $f(.)$, leaving the extremes values of $p_i$ out. The transformation of a single-peaked $f(.)$ is still single-peaked, so the analysis remains the same. The intensive versus extensive trade-off remains the same, and the solution has the same properties. In particular, the general insights from Propositions 1-3 continue to hold.

Introducing $\kappa$ as an additional parameter of the model allows us to run comparative statics with respect to accessibility of the media. The following is the main result of this section.

**Proposition 6.** *Suppose $f(.)$ is single-peaked. There exists some $\bar{\kappa} > 0$ such that for all $\kappa \in [0, \bar{\kappa}]$, $\sigma_0^*$ is decreasing in $\kappa$.*

*Proof.* See Appendix J. □

Proposition 6 should be read as: *If the captured media is less accessible to citizens, it is more informative in equilibrium.* The reason behind this is as follows: as $\kappa$ increases, the range of individuals who listen to the media shrinks. That is, given a media policy, $\overline{p}$ decreases and $\underline{p}$ increases. When $\kappa$ is small, however, $\overline{p} \approx 1$ and the extensive margin does not change significantly (the blue region in the Figure 1 remains almost the same). Since $\underline{p} >> 0$, the effect on the intensive margin is larger: the politician convinces much more people in the margin when she chooses a slightly more informative media (the red slice in the Figure 1 has higher density, because $\underline{p}$ increases). As a result, the trade-off leans towards the extensive margin, and the politician sets a more informative media.

Intuitively, with a cost of listening: only more moderate citizens (whose priors are around the critical decision cutoff) listen to the media, because they are the ones who benefit from the news most. But when the distribution is single-peaked, there are many of them, and they are very responsive to the media policy (in the sense that many people start listening even with slight improvements in the informativeness). For the politician, therefore, there are more people to gain in the extensive margin by setting a slightly more informative media, and the optimal decision represents this change in the trade-off.

With a single-dipped density, the symmetric result holds: an increase in $\kappa$ leads to an increase in $\sigma_1^*$, once again leading to an increase in informativeness of the captured media. The intuition is similar: a higher $\kappa$ implies extremist supporters listening to the media for a given policy. But then, the politician can more easily cater towards the other citizens by increasing the informativeness of media.

Proposition 6 suggests that a decrease in the accessibility of media may indeed be favorable for the citizens, because the moderate citizens will have access to a more informative media source. Most importantly, media capture may be more of a concern in environments where media is more easily accessible for citizens. The net welfare effects of an increase in the cost of media remains an open question.

# 7 Conclusion

This paper considers media capture as a strategic problem for the politician. On the technical front, it presents the first analysis of Bayesian persuasion towards an audience with heterogeneous priors. On the conceptual front, it extends out understanding of media capture by arguing when, and how, the informativeness of media is suppressed by a politician. In particular, media capture is a serious concern in environments where politician is more popular (Proposition 4), and in environments where media is more accessible for citizens (Proposition 6). Media capture can take the form of suppressing negative news (Proposition 2), positive news (Proposition 3), or suppressing the whole informative content of the news (Proposition 1) depending on the distribution of opinions in the society. Our model provides a framework for identifying these different cases and quantifying their implications.

On the practical front, our findings generate a group of interesting comparative statics. In addition to the comparative statics about the informativeness of the media with respect to different "types" of polarization of opinions (Proposition 5), a qualitative comparison of Proposition 2 and 3 suggests that, in societies with more extremists abundance of negative news may be preferable for the politician. These comparative statics yield testable implications, which can also be used to distinguish strategic and non-strategic models of media capture.

Our model is a static model with only one politician and one media source. Potential extensions with competition among politicians, with multiple media sources, and dynamic models with evolution of opinions will likely generate additional insights beyond the analysis considered here. Such extensions are left for future work.

# Appendix

## A Proof of Lemma 1

We will begin by considering a more general class of mechanisms than the one we consider in the main text, which are *persuasion mechanisms* as denoted by Kolotilin et al. (2017). In a persuasion mechanism, the politician is able to commit to a distinct signal distribution for each citizen. To this end, the politician asks each citizen to report her prior belief, and sends an action recommendation $m \in \{0, 1\}$ to the citizen[12] conditional on her report and the realized state.

A persuasion mechanism, therefore, is:

$$\psi := \{(\psi_0^i, \psi_1^i)\}_{i \in I}$$

where

$$\psi_0^i = Pr\{m = 0|\theta = 0\}$$
$$\psi_1^i = Pr\{m = 1|\theta = 1\}$$

are the probabilities of "personalized" action recommendations conditional on the reported *type* $p_i$ and the state $\theta$. The requirement to report the types truthfully and follow the action recommendation imposes an *incentive compatibility* requirement, which is formulated next.

Given a persuasion mechanism $\psi$, the subjective utility attained by citizen $i \in I$ if she reports type $p_j$ and takes actions $a_0 \in \{0, 1\}$ following $m = 0$, $a_1 \in \{0, 1\}$ following $m = 1$ is:

$$U_\psi(p_i, p_j, a_0, a_1) := (1 - p_i)(a_0(1 - \psi_0^j) + a_1\psi_0^j)(-c) + p_i(a_0(1 - \psi_1^j) + a_1\psi_1^j)(1 - c)$$

**Definition 1.** *A persuasion mechanism $\psi$ is* ***incentive compatible*** *if, for each $i \in I$:*

$$U_\psi(p_i) := U_\psi(p_i, p_i, 0, 1) \geq U_\psi(p_i, p_j, a_0, a_1) \quad \forall j \in I, a_0 \in \{0, 1\}, a_1 \in \{0, 1\}$$

The following result clarifies why the class of persuasion mechanisms are more general than the class of mechanisms considered in the main text.

**Lemma 5.** *Let $S$ be a set of signals, and assume the politician commit to some publicly observable signal distribution*

$$\sigma = (\sigma_0, \sigma_1) \quad \in \Delta(S) \times \Delta(S)$$

*The outcome of $\sigma$ can be implemented via a incentive compatible persuasion mechanism.*

[12] Due to the linearity of payoffs in actions, the citizen would prefer to have $a_i\{0, 1\}$ unless she has a knife-edge case of beliefs. In case of indifference, we will assume that the citizen will take $a = 1$, as the politician can always induce a slightly higher belief to break the indifference.

*Proof.* Fix $\sigma$. for any citizen $i \in I$, $\sigma$ generates two conditional distributions of posteriors at two states $\theta \in \{0,1\}$. Let the cdf of posterior distribution under $\theta = 0$ be $H^i_{\sigma,0}(.)$, and the cdf of posterior distribution under $\theta = 1$ be $H^i_{\sigma,1}(.)$. Then, set:

$$\psi^i_0 = 1 - H^i_{\sigma,0}(c)$$
$$\psi^i_1 = 1 - H^i_{\sigma,1}(c)$$

It is trivial to check that the persuasion mechanism $\{(\psi^i_0, \psi^i_1)\}_{i\in I}$ is incentive compatible. □

Given Lemma 5, we restrict attention to incentive compatible persuasion mechanisms. We will demonstrate that if $f(.)$ is single-peaked or single-dipped, the optimal strategy can be implemented by only two signals.

The following Lemma follows from usual arguments in mechanism design, and echoes Lemma 1 of Kolotilin et al. (2017).

**Lemma 6.** *For an incentive compatible persuasion mechanism $\psi$, the following conditions must be satisfied:*

1. $U'_\psi(p_i) = \psi^i_0 c + \psi^i_1(1-c)$ *for each* $p_i$,
2. $\psi^i_0 c + \psi^i_1(1-c)$ *is increasing in* $p_i$, *and,*
3. $U_\psi(0) = 0$ *and* $U_\psi(1) = 1 - c$.

*Proof.* The subjective payoff of citizen $i \in I$ if she reports truthfully and obeys the action recommendation is:

$$\begin{aligned} U_\psi(p_i) &:= (1-p_i)\psi^i_0(-c) + p_i\psi^i_1(1-c) \\ &= -\psi^i_0 c + (\psi^i_0 c + \psi^i_1(1-c))p_i \end{aligned}$$

One can then use the standard envelope arguments to derive that the first two conditions are necessary for $\psi$ to be incentive compatible.

By construction, $U_\psi(p_i)$ is bounded above by the payoff of "full disclosure" solution:

$$\overline{U}(p_i) = (1-c)p_i$$

and bounded below by the payoff of "no disclosure" solution:

$$\underline{U}(p_i) = \max\{p_i - c, 0\}$$

Consequently, one must have: $U_\psi(0) = 0$ and $U_\psi(1) = 1 - c$. □

The figure below illustrates the main idea of Lemma 6: incentive compatibility of $\psi$ restricts $U_\psi(p_i)$ to be an increasing and convex function between $\underline{U}(p_i)$ and $\overline{U}(p_i)$.

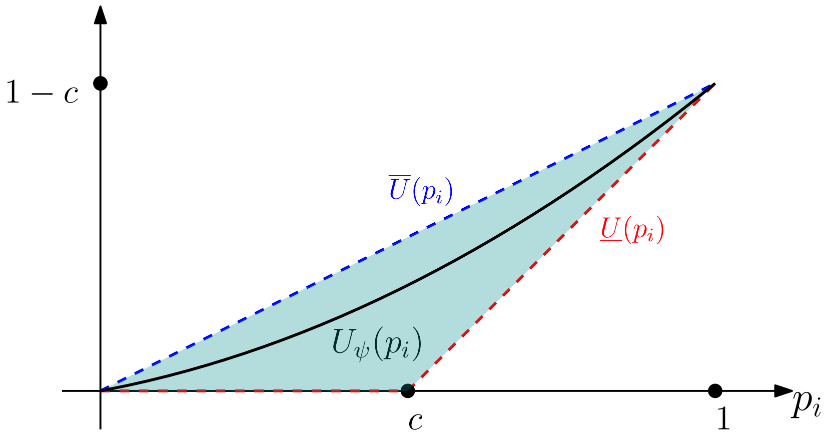


Figure 5: Illustration of Lemma 6.

**Lemma 7.** *If $f(.)$ is single-peaked, optimal incentive compatible persuasion mechanism $\psi^*$ induces a piecewise linear $U_{\psi^*}(p_i)$ as follows:*

$$U_{\psi^*}(p_i) = \begin{cases} 0, & \text{if } p_i < \underline{p} \\ \tau(p_i - \underline{p}), & \text{if } p_i \geq \underline{p} \end{cases}$$

*where $\tau^* \in [1-c, 1]$ and $\underline{p} = 1 - \frac{1-c}{\tau}$.*

*Proof.* The subjective payoff of politician under an incentive compatible persuasion mechanism $\psi$ is:

$$V(\psi) := \int_0^1 \left((1-p^*)\psi_0^i + p^*\psi_1^i\right) f(p_i)dp_i$$

Substituting $p^* = (1-c)$ and using the first part of Lemma 6 yields:

$$V(\psi) = \int_0^1 \left(c\psi_0^i + (1-c)\psi_1^i\right) f(p_i)dp_i = \int_0^1 U'_\psi(p_i)f(p_i)dp_i$$

Integration by parts and using the last part of Lemma 6 gives:

$$V(\psi) = (1-c)f(1) - \int_0^1 U_\psi(p_i)f'(p_i)dp_i$$

Therefore, the optimal incentive compatible persuasion mechanism must be the solution to:

$$\max_\psi - \int_0^1 U_\psi(p_i)f'(p_i)dp_i$$

subject to:

$$U_\psi(p_i) \text{ is increasing and convex,}$$
$$\underline{U}(p_i) \leq U_\psi(p_i) \leq \overline{U}(p_i)$$

Since $f(.)$ is single-peaked and continuously differentiable, $f'(.)$ is continuous and crosses zero once from above. Let $\tilde{p}$ be the point where $f'(\tilde{p}) = 0$. Then, politician wants to minimize $U_\psi(p_i)$ for $p_i < \tilde{p}$ and maximize $U_\psi(p_i)$ for $p_i \geq \tilde{p}$. One can then replicate the argument in Theorem 2 of Kolotilin et al. (2017) to prove that a piecewise linear payoff schedule improves upon any other increasing and convex payoff schedule. Below is an illustration of the idea:

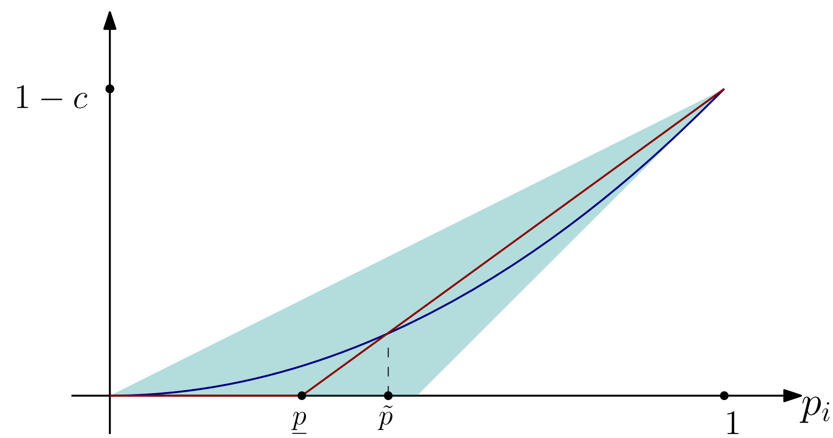


Figure 6: Optimality of a piecewise linear function.

Here, the blue curve is the increasing and convex payoff schedule, and the red curve is the piecewise linear function that improves upon it. □

**Lemma 8.** *If $f(.)$ is single-peaked, the payoff schedule of optimal incentive compatible persuasion mechanism $U_{\psi^*}(p_i)$ can be implemented by a publicly observable signal distribution with only two signals.*

*Proof.* By Lemma 7, $U_{\psi^*}(p_i)$ is piecewise and linear with a kink at $\underline{p}$. One can then take a publicly observable signal distribution

$$\sigma = (\sigma_0, \sigma_1) \in [0, 1] \times [0, 1]$$

and set $\sigma_1 = 1$ and $\sigma_0 = \frac{\tau-(1-c)}{c}$. It is trivial to check that this signal distribution yields the same payoff schedule. □

For a single-dipped $f(.)$, one can use the symmetric arguments to replicate Lemma 7 and 8.

# B Simplification of Politician's Problem

This section involves more detailed steps for the simplification of politician's problem in Section 3.1.

Substituting in $p^* = 1 - c$ into the objective function of politician and rearranging gives:

$$\left(F\left(\frac{c(1-\sigma_0)}{c(1-\sigma_0)+(1-c)(1-\sigma_1)}\right)-F\left(\frac{c\sigma_0}{c\sigma_0+(1-c)\sigma_1}\right)\right)(p^*\sigma_1+(1-p^*)\sigma_0)+\left(1-F\left(\frac{c(1-\sigma_0)}{c(1-\sigma_0)+(1-c)(1-\sigma_1)}\right)\right)$$
$$=\left(F\left(\frac{c(1-\sigma_0)}{c(1-\sigma_0)+(1-c)(1-\sigma_1)}\right)-F\left(\frac{c\sigma_0}{c\sigma_0+(1-c)\sigma_1}\right)\right)((1-c)\sigma_1+c\sigma_0)+\left(1-F\left(\frac{c(1-\sigma_0)}{c(1-\sigma_0)+(1-c)(1-\sigma_1)}\right)\right)$$
$$=F\left(\frac{c(1-\sigma_0)}{c(1-\sigma_0)+(1-c)(1-\sigma_1)}\right)(c(-1+\sigma_0)+(1-c)(-1+\sigma_1))-F\left(\frac{c\sigma_0}{c\sigma_0+(1-c)\sigma_1}\right)(c\sigma_0+)(1-c)\sigma_1)+1$$

Once again rearranging, removing the constant term which does not affect the optimal value of the decision variable, and turning this into a minimization instead of a maximization problem, one can write the politician's problem as the one in Equation 1.

# C Proof of Lemma 3

*Proof.* Assume, to get a contradiction, that there is an interior solution $(\sigma_0^*, \sigma_1^*) \in int(\Delta)$, which implies:

$$\frac{\partial \Pi(F,(\sigma_0^*,\sigma_1^*))}{\partial \sigma_0}=0$$
$$\frac{\partial \Pi(F,(\sigma_0^*,\sigma_1^*))}{\partial \sigma_1}=0$$

Taking derivatives of $\Pi$ yields:

$$\frac{\partial \Pi(F,(\sigma_0,\sigma_1))}{\partial \sigma_0}=c\left[f\left(\frac{c\sigma_0}{c\sigma_0+(1-c)\sigma_1}\right)\frac{(1-c)\sigma_1}{c\sigma_0+(1-c)\sigma_1}-f\left(\frac{c(1-\sigma_0)}{c(1-\sigma_0)+(1-c)(1-\sigma_1)}\right)\frac{(1-c)(1-\sigma_1)}{c(1-\sigma_0)+(1-c)(1-\sigma_1)}\right]$$
$$-c\left[\left(F\left(\frac{c(1-\sigma_0)}{c(1-\sigma_0)+(1-c)(1-\sigma_1)}\right)-F\left(\frac{c\sigma_0}{c\sigma_0+(1-c)\sigma_1}\right)\right)\right]$$
$$\frac{\partial \Pi(F,(\sigma_0,\sigma_1))}{\partial \sigma_1}=(1-c)\left[f\left(\frac{c(1-\sigma_0)}{c(1-\sigma_0)+(1-c)(1-\sigma_1)}\right)\frac{c(1-\sigma_0)}{c(1-\sigma_0)+(1-c)(1-\sigma_1)}-f\left(\frac{c\sigma_0}{c\sigma_0+(1-c)\sigma_1}\right)\frac{c\sigma_0}{c\sigma_0+(1-c)\sigma_1}\right]$$
$$-(1-c)\left[\left(F\left(\frac{c(1-\sigma_0)}{c(1-\sigma_0)+(1-c)(1-\sigma_1)}\right)-F\left(\frac{c\sigma_0}{c\sigma_0+(1-c)\sigma_1}\right)\right)\right]$$

Substituting $\underline{p}$ and $\bar{p}$ from Lemma 2, this simplifies to:

$$\frac{\partial \Pi(F,.)}{\partial \sigma_0}=c\left[f\left(\underline{p}\right)(1-\underline{p})-f\left(\bar{p}\right)(1-\bar{p})-\left(F\left(\bar{p}\right)-F\left(\underline{p}\right)\right)\right]$$
$$\frac{\partial \Pi(F,.)}{\partial \sigma_1}=(1-c)\left[f\left(\bar{p}\right)\bar{p}-f\left(\underline{p}\right)\underline{p}-\left(F\left(\bar{p}\right)-F\left(\underline{p}\right)\right)\right]$$

Since $c \in (0,1)$, in the optimal solution, one has:

$$F\left(\bar{p}\right)-F\left(\underline{p}\right)=f\left(\underline{p}\right)(1-\underline{p})-f\left(\bar{p}\right)(1-\bar{p})$$
$$F\left(\bar{p}\right)-F\left(\underline{p}\right)=f\left(\bar{p}\right)\bar{p}-f\left(\underline{p}\right)\underline{p}$$

These equalities can be simultaneously satisfied under two possible scenarios: (i) $\underline{p}=\bar{p}$, or (ii) $f(\underline{p})=f(\bar{p})$. Case (i) corresponds to the $\sigma_0=\sigma_1$ case, which contradicts our initial assumption that the solution is interior. Therefore, case (ii) must be true. But then the following must hold in the optimal solution:

$$\left(F\left(\bar{p}\right) - F\left(\underline{p}\right)\right) = f(\bar{p})(\bar{p} - \underline{p})$$

which can be rewritten as:

$$\int_{\underline{p}}^{\bar{p}} f(s)ds = \int_{\underline{p}}^{\bar{p}} f(\bar{p})ds$$

thus

$$\int_{\underline{p}}^{\bar{p}} f(s)ds = \int_{\underline{p}}^{\bar{p}} f(\bar{p})ds$$

where $f(\bar{p}) = f(\underline{p})$. It's easy to see that, for any single-peaked or single-dipped $f(.)$, this solution cannot be satisfied for $\underline{p} \neq \bar{p}$.[13] This is because for a single-peaked $f(.)$, we have: $f(s) > f(\bar{p})$ for all $s \in (\underline{p}, \bar{p})$. Similarly, for a single-dipped $f(.)$, $f(s) < f(\bar{p})$ for all $s \in (\underline{p}, \bar{p})$. The contradiction follows. □

# D Checking the Boundaries of $\Delta$

Lemma 3 demonstrates that the optimal solution must be in the boundaries. The remaining question is: which boundary? We apply a further change of variables into the problem which will allow us to redefine the problem as an optimization problem with one variable only.

1. Begin with the $\sigma_0 = \sigma_1$ boundary.The value of objective function in Equation (1) in this boundary is $F(c)$ for $\sigma_0 \in [0, 1]$.

2. Now, consider the $\sigma_0 = 0$ boundary. The value of objective function in this boundary is:

$$\Pi(F, (0, \sigma_1)) = F\left(\frac{c}{c + (1-c)(1-\sigma_1)}\right)(c + (1-c)\sigma_1)$$

   for $\sigma_1 \in [0, 1]$. Adopting the transformation $z := \frac{c}{c+(1-c)(1-\sigma_1)}$ suggests that this can be written as: $\frac{F(z)}{z}c$ where $z$ varies between $c$ and 1. Note that Equation 1 is a minimization problem, so essentially we will calculate

$$\min_{z \in [c,1]} \frac{F(z)}{z} c$$

   and compare it to the minimum values obtained in other boundaries.

3. Finally, consider the $\sigma_1 = 1$ boundary. The value of objective function in this boundary is:

$$\Pi(F, (\sigma_0, 1)) = c(1 - \sigma_0) + F\left(\frac{c\sigma_0}{c\sigma_0 + 1 - c}\right)(c\sigma_0 + 1 - c)$$

[13] Just drawing $f(.)$ would provide a sufficient visual interpretation.

for $\sigma_0 \in [0,1]$. Adopting the transformation $z := \frac{c\sigma_0}{c\sigma_0+1-c}$, this can be written as: $1 - \frac{1-F(z)}{1-z}(1-c)$ where $z$ varies between 0 and $c$. Therefore, in this boundary, we will calculate

$$\min_{z\in[0,c]} 1 - \frac{1-F(z)}{1-z}(1-c)$$

and compare it to the minimum values obtained in other boundaries.

All in all, the politician's problem can be reduced to:

$$\min\left\{F(c), \min_{z\in[c,1]} \frac{F(z)}{z}c, \min_{z\in[0,c]} 1 - \frac{1-F(z)}{1-z}(1-c)\right\} \tag{2}$$

Finally, one can see that $F(c)$ is indeed the "corner case" for either of the functions in the minimization problems (plugging $z = c$ into either function yields $F(c)$), hence it is redundant. Therefore, the problem we will consider is:

$$\min\{\min_{z\in[c,1]} \frac{F(z)}{z}c, \min_{z\in[0,c]} 1 - \frac{1-F(z)}{1-z}(1-c)\}$$

or, equivalently,

$$\min_{z\in[0,1]} G(z) \tag{3}$$

where

$$G(z) = \begin{cases} 1 - \frac{1-F(z)}{1-z}(1-c) & \text{if } z \leq c \\ \frac{F(z)}{z}c & \text{if } z \geq c \end{cases}$$

This is a straightforward, one-dimensional optimization problem. The interpretation of solution and its mapping to the politician's original problem is as follows.

- If the minimizer of this problem is $z = c$, the solution has $\sigma_0 = \sigma_1$, i.e. it is fully uninformative.
- If the minimizer of this problem is some $z \in (c, 1)$, then the solution has $\sigma_0 = 0$, i.e. the "good" message is fully informative.
- If the minimizer of this problem is some $z \in (0, c)$, then the solution has $\sigma_1 = 1$, i.e. the "bad" message is fully informative.
- As an extreme case, if some $z \in \{0, 1\}$ is the minimizer,[14] then the solution has $\sigma_0 = 0$ and $\sigma_1 = 1$, i.e. the messages are fully informative.

# E Proof of Proposition 1

*Proof.* Consider the first case in the proposition. Note that if $f(.)$ is increasing, than $F(.)$, being its integral, must be convex in $[0, 1]$. This, in particular, implies that: (i) $\frac{F(z)}{z}$ is

[14]One can check that $G(0) = G(1)$, so if one is a minimizer, the other one is a minimizer too.

increasing everywhere, and (ii) $\frac{1-F(z)}{1-z}$ is increasing everywhere. Therefore, we have:

$$\arg\min_{z\in[c,1]} \frac{F(z)}{z} = c$$

and

$$\arg\min_{z\in[0,c]} -\frac{1-F(z)}{1-z} = c$$

hence $\arg\min_{z\in[0,1]} G(z) = c$, which corresponds to the fully uninformative solution.

Now, consider the second case in the proposition. Note that if $f(.)$ is decreasing, than $F(.)$ must be concave in $[0,1]$. This, in particular, implies that: (i) $\frac{F(z)}{z}$ is decreasing everywhere, and (ii) $\frac{1-F(z)}{1-z}$ is decreasing everywhere. Therefore, we have:

$$\arg\min_{z\in[c,1]} \frac{F(z)}{z} = 1$$

and

$$\arg\min_{z\in[0,c]} -\frac{1-F(z)}{1-z} = 0$$

hence $\arg\min_{z\in[0,1]} G(z) = \{0,1\}$, which corresponds to the fully informative solution. □

# F Proof of Proposition 2

*Proof.* Remember the program introduced given by Equation (3), and denote the minimizer by $z^*$:

$$z^* := \arg\min_{z\in[0,1]} G(z)$$

where

$$G(z) = \begin{cases} 1 - \frac{1-F(z)}{1-z}(1-c) & z \leq c \\ \frac{F(z)}{z}c & z \geq c \end{cases}$$

Recall, for reference, that $z^* = c$ corresponds to the fully uninformative solution, and $z^* \in \{0,1\}$ corresponds to the the fully informative solution. Most of the argument would include ruling out $c$ or $\{0,1\}$ as potential values of $z^*$.

1. Begin with the first part of Proposition, where $F(c) > c$. We begin by showing that in this case the optimal solution cannot be fully uninformative, i.e. we cannot have $z^* = c$. This is rather easy to see: note that $F(1) = 1$ by construction, so

$$\frac{F(1)}{1}c < F(c)$$

That is, $G(1) < G(c)$, so one cannot have $z^* = c$. Now we continue by showing that we must have $\sigma_1 = 1$ in the optimal solution. This corresponds to ruling out $z^* \in (c, 1)$, established by the following Lemma.

**Lemma 9.** *Suppose $f(.)$ is continuous and single-peaked, and $F(c) > c$. Then, for any $z \in (c, 1)$, $F(z) > z$.*

*Proof.* Suppose, to get a contradiction, that $F(z^*) - z^* \leq 0$ for some $z^* \in (c, 1)$. Combined with the fact that $\mathbb{F}(c) - c > 0$, one can invoke the Mean Value Theorem to conclude:

$$\text{There exists some } z' \in [c, z^*] \text{ such that } f(z') - 1 < 0.$$

Similarly, combining $\mathbb{F}(z^*) - z^* \leq 0$ with the fact that $F(1) = 1$, one can invoke the Mean Value Theorem once more to conclude:

$$\text{There exists some } z'' \in [z^*, 1] \text{ such that } f(z'') - 1 \geq 0.$$

Note that $f(.)$ is single-peaked. Given that $f(z'') > f(z')$, we conclude that the peak of $f(.)$ must lie to the right of $z''$. In particular, this implies that $f(.)$ will be increasing in $[0, z'']$, and consequently $F(.)$ must be convex and $\frac{F(z)}{z}$ must be increasing in this region. But note that we have $0 < c < z^* \leq z''$ while:

$$\frac{F(c)}{c} > 1 \geq \frac{F(z^*)}{z^*}$$

a contradiction. □

Lemma 9 establishes that $\frac{F(z)}{z}c > \frac{F(1)}{1}c$ for all $z \in [c, 1)$, and therefore one cannot have $z^* \in [c, 1)$ in the optimal solution. As a result, one cannot have a solution with $\sigma_1 < 1$. We conclude that if the conditions in the first part of Proposition 2 are satisfied, one must have: $z^* \in [0, c) \cup \{1\}$, which corresponds to: $\sigma_1 = 1$ in the optimal solution. Tighter characterization of $\sigma_0$ is not possible, as for any $x \in [0, c]$, one can come up with a single-peaked distribution which has $\sigma_0 = x$ in the optimal solution. In particular, one can take $F(.)$ as the step function,[15] where the step is at $q = \frac{x}{1+x} \in [0, c]$. Based on Kamenica and Gentzkow (2011), we know that the optimal solution must have $\sigma_0 = \frac{q}{1-q} = x$.

2. Now, consider the second part of Proposition, where $F(c) < c$. We begin by showing that in this case the optimal solution cannot be fully informative, i.e. we cannot have $z^* \in \{0, 1\}$. This is also easy to see: because

$$\frac{F(c)}{c} < 1 = \frac{F(1)}{1}$$

[15] To preserve the continuity of $F(.)$, one can consider an arbitrarily close approximation to the step function.

one has $G(c) < G(1) = G(0)$, so one cannot have $z^* \in \{0, 1\}$. We continue by showing that one must have $\sigma_1 = 1$ in the optimal solution, which again corresponds to ruling out $z^* \in (c, 1)$. The following Lemma proves this.

**Lemma 10.** *Suppose $f(.)$ is continuous and single-peaked, and $F(c) < c$. Then, for any $z \in (c, 1)$, $\frac{F(z)}{z} \geq \frac{F(c)}{c}$.*

*Proof.* Suppose, to get a contradiction, that $\frac{F(z^*)}{z^*} < \frac{F(c)}{c}$ for some $z^* \in (c, 1)$. Invoking the Mean Value Theorem three times, we have:

- There exists a $z' \in [0, c]$ such that $f(z') = \frac{F(c)}{c}$.
- There exists a $z'' \in [c, z^*]$ such that $f(z'') = \frac{F(z^*)-F(c)}{z^*-c}$.
- There exists a $z''' \in [z^*, 1]$ such that $f(z''') = \frac{1-F(z^*)}{1-z^*}$.

But note that, since $\frac{F(z^*)}{z^*} < \frac{F(c)}{c}$:

$$\frac{F(z^*) - F(c)}{z^* - c} < \frac{F(c)}{c}$$

Similarly, since $\frac{F(z^*)}{z^*} < \frac{F(c)}{c} < 1$:

$$\frac{1 - F(z^*)}{1 - z^*} > \frac{F(c)}{c}$$

Putting everything together:

$$f(z''') > f(z') > f(z'')$$

whereas $z' < z'' < z'''$, which contradicts $f(.)$ being single-peaked. □

By Lemma 10, one cannot have $z^* \in (\frac{1}{2}, 1]$ in the optimal solution; therefore, one cannot have a solution with $\sigma_1 < 1$ (unless $\sigma_0 = \sigma_1$, in which case the politician is indifferent among any value of $\sigma_0$).

In this case, tighter characterization of $\sigma_0$ is indeed possible, as $\frac{F(c)}{c}$ constitutes a natural lower bound on the value of objective function. We will begin by showing that in equilibrium, this translates into a lower bound on $z^*$.

**Lemma 11.** *Suppose $f(.)$ is continuous and single-peaked, and $F(c) < c$. Then $z^* \in [\underline{z}, c]$ where*

$$\underline{z} = \frac{c - F(c)}{1 - F(c)}$$

*Proof.* Lemma 10 and the discussion preceding it already establishes that $z^* \notin (c, 1]$. We will show that $z^* \notin [0, \underline{z})$.

Take any $z \in [0, \underline{z})$. All we need to show is that $G(z) > G(c)$, i.e.

$$\frac{1 - F(z)}{1 - z} < \frac{1 - F(c)}{1 - c}$$

This easily follows from

$$\frac{1 - F(z)}{1 - z} \leq \frac{1}{1 - z} < \frac{1}{1 - \underline{z}} = \frac{1 - F(c)}{1 - c}$$

where the first inequality holds because $F(z) \geq 1$, the second one holds because $z < \underline{z}$, and the equality holds by construction of $\underline{z}$. □

Lemma 11 imposes a lower bound on $z^*$ in the region where $z \in [0, c]$. Recall that this is the region where $\sigma_1 = 1$ and $z = \frac{c\sigma_0}{c\sigma_0 + (1-c)} \Leftrightarrow \sigma_0 = \frac{z(1-c)}{c(1-z)}$, so the lower bound on $z^*$ implies a lower bound on $\sigma_0^*$. The lower bound turns out to be:

$$\underline{\sigma}_0^* = \frac{\underline{z}(1 - c)}{c(1 - \underline{z})} = \frac{c - F(c)}{c}$$

and the result follows.

□

## G Proof of Proposition 3

*Proof.* The idea of this proof is quite similar to that of Proposition 2, so we will be brief and lay out the basics.

1. Begin with the case $F(c) > c$. Because $G(0) = G(1) < G(c)$, we conclude that the solution can never be fully uninformative.

   Next, we rule out the case that $z^* \in (0, c]$. Once again, repeatedly using Mean Value Theorem gives:

   $$\text{There exists } z' \in [0, z^*],\ z'' \in [z^*, c],\ z''' \in [c, 1] \text{ such that}$$
   $$f(z') < f(z'') \text{ and } f(z''') < f(z'').$$

   But this contradicts $f(.)$ being single-dipped.

   Finally, the lower bound for $\sigma_1$ follows from combining two facts: (i) in the optimal solution $z^* \in \{0\} \cup (c, 1]$, one needs to have $\frac{F(z^*)}{z^*} \leq 1$, and (ii) we must have: $F(z^*) \geq F(c)$.

2. Now, consider the case $F(c) < c$. Since $G(c) < G(0) = G(1)$, one cannot have $z^* \in \{0, 1\}$.

   In order to rule out $z^* \in (0, c)$, suppose the contrary, i.e. $z^* \in (0, c)$. One can show that:

   $$\text{There exists } z' \in [0, z^*] \text{ and } z'' \in [z^*, c] \text{ such that } f(z'') > f(z').$$

   But the single-dippedness of $f(.)$ then suggests that $F(.)$ must be convex in $[z^*, 1]$. But this implies that $F(z)$ must hit the value of 1 at some $z < 1$, which contradicts $f(.)$ being single-dipped.

□

# H Proof of Proposition 4

*Proof.* **of Part (a).** By Proposition 2, the optimal solution will have $\sigma_1^* = \tilde{\sigma}_1^* = 1$. So it is sufficient to consider the following program introduced in Equation (3):

$$\min_{z \in [0,c]} 1 - \frac{1 - F(z)}{1 - z}(1 - c),$$

where $z = \frac{c\sigma_0}{c\sigma_0 + 1 - c}$. This is equivalent to

$$\max_{z \in [0,c]} \log(1 - F(z)) - \log(1 - z).$$

Define

$$\phi(z) = -\frac{f(z)}{1 - F(z)} + \frac{1}{1 - z}.$$

The first-order condition requires that $\phi(z^*) = 0$, and the second-order condition requires that $\phi'(z^*) < 0$, so that $\phi(z^*)$ crosses 0 from above. By the MLRP, $\frac{f(z)}{1-F(z)} < \frac{\tilde{f}(z)}{1-\tilde{F}(z)}$, so that $\phi(z) > \tilde{\phi}(z)$ for all $z$. But since $\phi(z^*)$ and $\tilde{\phi}(\tilde{z}^*)$ cross 0 from above, this means that $z^* > \tilde{z}^*$ and therefore $\sigma_0^* > \tilde{\sigma}_0^*$.

**Proof of Part (b).** The argument is along similar lines as in the proof of part (a). By Proposition 3, at the optimum $\sigma_0^* = \tilde{\sigma}_0^* = 0$ and so we can consider the following program:

$$\min_{z \in [c,1]} \frac{F(z)}{z} c,$$

where $z := \frac{c}{c + (1-c)(1-\sigma_1)}$. The program is equivalent to

$$\min_{z \in [c,1]} \log(F(z)) - \log(z).$$

Define

$$\phi(z) = \frac{f(z)}{F(z)} - \frac{1}{z}.$$

The first-order condition requires that $\phi(z^*) = 0$, and the second-order condition requires that $\phi'(z^*) > 0$, so that $\phi(z^*)$ crosses 0 from below. By the MLRP, $\frac{f(z)}{F(z)} > \frac{\tilde{f}(z)}{\tilde{F}(z)}$, so that $\phi(z) > \tilde{\phi}(z)$ for all $z$. But since $\phi(z^*)$ and $\tilde{\phi}(\tilde{z}^*)$ cross 0 from below, this means that $z^* < \tilde{z}^*$ and therefore $\sigma_1^* < \tilde{\sigma}_1^*$. □

# I Proof of Proposition 5

*Proof.* By Lemma 9, $\frac{S(z)}{z} > \frac{S(1)}{1} = 1$ for all $z \in (c, 1)$. Therefore, $S(z) > z$ for all $z \in (c, 1)$. This immediately implies that $F_{\alpha,\rho}(z) > z$ for all $z \in (c, 1)$. So to find the optimal $\sigma_0$ and $\sigma_1$ when the distribution of the priors is given by $F_{\alpha,\rho}$ it is sufficient to consider the following program:

$$\min_{z \in [0,c]} 1 - \frac{1 - F_{\alpha,\rho}(z)}{1 - z}(1 - c),$$

where

$$F_{\alpha,\rho}(z) = \left(\alpha S(z)^\rho + (1 - \alpha) z^\rho\right)^{\frac{1}{\rho}}.$$

Define $T(z) = S(z)/z$. Then the program can be written as

$$\max_{z \in [0,c]} \frac{1 - z\left(\alpha T(z)^\rho + 1 - \alpha\right)^{\frac{1}{\rho}}}{1 - z},$$

The first-order condition is given by

$$z^* \alpha T'(z^*) T(z^*)^{\rho-1}(-1 + z^*)(\alpha T(z^*)^\rho + 1 - \alpha)^{(1-\rho)/\rho} - (\alpha T(z^*)^\rho + 1 - \alpha)^{1/\rho} + 1 = 0.$$

This equation can be solved for $T'(z^*)$ to get

$$\begin{aligned} T'(z^*) &= \frac{1 - (\alpha T(z^*)^\rho + 1 - \alpha)^{1/\rho}}{z^*(1 - z^*)\alpha T(z^*)^{\rho-1}(\alpha T(z^*)^\rho + 1 - \alpha)^{-(\rho-1)/\rho}} \\ &= \frac{\omega^{1-\frac{1}{\rho}} - \omega}{z^*(1 - z^*)\alpha T(z^*)^{\rho-1}}. \end{aligned}$$

where $\omega = \alpha T(z^*)^\rho + 1 - \alpha \in [0, 1]$. Next note that the above program is equivalent to

$$\max_{z \in [0, \frac{1}{2}]} \Omega(z, \alpha),$$

where $\Omega(z, \alpha) = \log(1 - z\left(\alpha T(z)^\rho + 1 - \alpha\right)^{\frac{1}{\rho}}) - \log(1 - z)$. Some simple algebra results in

$$\operatorname{sign}\left(\frac{\partial^2 \Omega(z^*, \alpha)}{\partial z \partial \alpha}\right) = \operatorname{sign}\left(-\alpha(z^* T'(z^*) + T(z^*)) T(z^*)^{2\rho} + ((\alpha + (z^* \omega^{1/\rho} - 1)\rho) z^* T'(z^*) + T(z^*)(2\alpha - 1)) T(z^*)^\rho - T(z^*)(\alpha - 1)\right).$$

Using the expression for $T'(z^*)$:

$$\begin{aligned}\text{sign}\left(\frac{\partial^2\Omega(z^*,\alpha)}{\partial z\partial\alpha}\right) &= \text{sign}\left(\frac{(\alpha T(z^*)^\rho + 1-\alpha)^{\frac{1}{\rho}}\rho - \alpha T(z)^\rho - \rho + \alpha}{\rho}\right) \\ &= \text{sign}\left(\frac{1-\omega}{\rho} - (1-\omega^{1/\rho})\right) \\ &= \begin{cases} 1 & \text{if} \quad \rho < 1, \\ -1 & \text{if} \quad \rho > 1, \\ 0 & \text{if} \quad \rho = 1. \end{cases}\end{aligned}$$

The first-order condition is given by $\Omega(z^*,\alpha) = 0$. By the implicit function theorem,

$$\frac{\partial z^*}{\partial\alpha} = \frac{-\partial\Omega(z^*,\alpha)/\partial\alpha}{\partial\Omega(z^*,\alpha)/\partial z}.$$

But the second-order condition is given by $\partial\Omega(z^*,\alpha)/\partial z < 0$. Therefore, the sign of $\frac{\partial z^*}{\partial\alpha}$ is the same as the sign of $\frac{\partial\Omega(z^*,\alpha)}{\partial\alpha}$. This proves that

$$\text{sign}\left(\frac{\partial z^*}{\partial\alpha}\right) = \begin{cases} 1 & \text{if} \quad \rho < 1, \\ -1 & \text{if} \quad \rho > 1, \\ 0 & \text{if} \quad \rho = 1, \end{cases}$$

and therefore

$$\text{sign}\left(\frac{\partial\sigma_0^*}{\partial\alpha}\right) = \begin{cases} 1 & \text{if} \quad \rho < 1, \\ -1 & \text{if} \quad \rho > 1, \\ 0 & \text{if} \quad \rho = 1. \end{cases}$$

□

# J Proof of Proposition 6

*Proof.* Using the same arguments as in the proof of Lemma 3 and Proposition 2, one can demonstrate that that the optimal solution has: $\sigma_1^* = 1$ and $\sigma_0^* \in [0,1]$. Therefore, in the optimal solution:

$$\begin{aligned}\bar{p}(\sigma_0,\kappa) &= 1 - \frac{\kappa}{c(1-\sigma_0)} \\ \underline{p}(\sigma_0,\kappa) &= \frac{c\sigma_0 + \kappa}{c\sigma_0 + (1-c)}\end{aligned}$$

Given $\kappa$, the politician's problem then is reduced to the following program:

$$\max_{\sigma_0\in[0,1]} \Pi(\sigma_0,\kappa)$$

where

$$\Pi(\sigma_0, \kappa) = -F(\bar{p}(\sigma_0, \kappa))c(1-\sigma_0) - F(\underline{p}(\sigma_0, \kappa))\left(c\sigma_0 + (1-c)\right)$$

The first-order condition is:

$$\frac{\partial \Pi(\sigma_0, \kappa)}{\partial \sigma_0} = c\left(f(\bar{p}(\sigma_0, \kappa))(1-\bar{p}(\sigma_0, \kappa)) - f(\underline{p}(\sigma_0, \kappa))(1-\underline{p}(\sigma_0, \kappa)) + F(\bar{p}(\sigma_0, \kappa)) - F(\underline{p}(\sigma_0, \kappa))\right) = 0$$

Let

$$\Lambda(\sigma_0, \kappa) := c\left(f(\bar{p}(\sigma_0, \kappa))(1-\bar{p}(\sigma_0, \kappa)) - f(\underline{p}(\sigma_0, \kappa))(1-\underline{p}(\sigma_0, \kappa)) + F(\bar{p}(\sigma_0, \kappa)) - F(\underline{p}(\sigma_0, \kappa))\right)$$

The first-order condition then is: $\Lambda(\sigma_0^*, \kappa) = 0$. By the Implicit Function Theorem,

$$\frac{\partial \sigma_0^*}{\partial \kappa} = -\frac{\partial \Lambda(\sigma_0^*, \kappa)/\partial \kappa}{\partial \Lambda(\sigma_0^*, \kappa)/\partial \sigma_0}$$

But the second-order condition is that $\frac{\partial \Lambda(\sigma_0^*, \kappa)}{\partial \sigma_0} < 0$. Therefore, the sign of $\frac{\partial \sigma_0^*}{\partial \kappa}$ is the same as the sign of $\frac{\partial \Lambda(\sigma_0^*, \kappa)}{\partial \kappa}$. Simple algebra gives:

$$\frac{\partial \Lambda(\sigma_0^*, \kappa)}{\partial \kappa} = c\left(f'(\overline{p}(\sigma_0^*, \kappa))\frac{\partial \overline{p}(\sigma_0^*, \kappa)}{\partial \kappa}(1-\overline{p}(\sigma_0^*, \kappa)) - \underbrace{f'(\underline{p}(\sigma_0^*, \kappa))}_{>0}\underbrace{\frac{\partial \underline{p}(\sigma_0^*, \kappa)}{\partial \kappa}}_{>0}\underbrace{(1-\underline{p}(\sigma_0^*, \kappa))}_{>0}\right)$$

Note that when $\kappa = 0$, $\overline{p}(\sigma_0^*, \kappa) = 1$ and $\frac{\partial \Lambda(\sigma_0^*, \kappa)}{\partial \kappa} < 0$. By Berge's maximum theorem, $\overline{p}(\sigma_0^*, \kappa)$ is continuous in $\kappa$. As a result, there exists some $\overline{\kappa} > 0$ such that, for $\kappa \in (0, \overline{\kappa})$, $\frac{\partial \Lambda(\sigma_0^*, \kappa)}{\partial \kappa} < 0$. The result follows. □

# References


**Besley, Timothy, and Andrea Prat.** 2006. "Handcuffs for the Grabbing Hand? Media Capture and Government Accountability." *The American Economic Review*, 96(3): 720–736.

**Gehlbach, Scott, and Konstantin Sonin.** 2014. "Government Control of the Media." *Journal of Public Economics*, 118 163–171.

**Gentzkow, Matthew.** 2016. "Polarization in 2016." *Toulouse Network of Information Technology White Paper.*

**Kamenica, Emir, and Matthew Gentzkow.** 2011. "Bayesian Persuasion." *The American Economic Review*, 101(6): 2590–2615.

**Kolotilin, Anton.** 2015. "Experimental design to persuade." *Games and Economic Behavior*, 90 215–226.

**Kolotilin, Anton.** 2018. "Optimal Information Disclosure: A Linear Programming Approach." *Theoretical Economics*, 13(2): 607–635.

**Kolotilin, Anton, Tymofiy Mylovanov, Andriy Zapechelnyuk, and Ming Li.** 2017. "Persuasion of a Privately Informed Receiver." *Econometrica*, 85(6): 1949–1964.

**Rayo, Luis, and Ilya Segal.** 2010. "Optimal Information Disclosure." *Journal of political Economy*, 118(5): 949–987.

**Sunstein, Cass.** 2001. *Republic. com.*: Princeton University Press Princeton, NJ.