Expectations and the Term Structure of Interest Rates*

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Abstract

This paper studies the relationship between investors' subjective expectations and the term structure of interest rates. Departing from rational expectations, we allow investors to hold arbitrary—and potentially heterogeneous—beliefs about future interest rates. We derive the relationships that expected and realized interest rates must satisfy under different assumptions about expectation formation, and we develop regression-based tests for two key hypotheses: (i) that bond risk premia are constant, and (ii) that investors' expectations across maturities and forecast horizons are consistent with one another. Using survey data, we find no evidence of time-varying risk premia for short-term bonds. We also document that market participants' expectations are inconsistent with the structural relationships that link short- and long-term interest rates.

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1 Introduction

According to the expectations hypothesis of the term structure of interest rates (EH), the yield on a long-term bond equals the average of expected short-term interest rates over the bond's lifetime, plus a constant risk premium. That is, if $y_t^{(n)}$ is the yield on a bond that matures in n periods, then

$$y_t^{(n)} = \frac{1}{n}\sum_{s=0}^{n-1}\mathbb{E}_t^*[y_{t+s}^{(1)}] + \text{constant}, \tag{EH}$$

where $\mathbb{E}^*[\cdot]$ denotes expectations under the true data-generating process. This expression is just one of several algebraically equivalent formulations of the expectations hypothesis. For instance, the hypothesis also implies that forward rates should equal expected future short-term rates (up to a constant risk premium), and expected holding-period returns should be identical across maturities (again up to a constant).

A large body of empirical research has consistently documented evidence against various formulations of the expectations hypothesis. For example, Campbell and Shiller (1991) document that (i) excess returns are not equalized across maturities, and (ii) an upward-sloping yield curve is not associated with a corresponding increase in future short-term interest rates. The standard interpretation of these findings is that the expectations hypothesis fails due to the presence of time-varying risk premia.

At the same time, it has long been recognized that standard tests of the expectations hypothesis—such as the regressions mentioned above—are in fact *joint tests* of the expectations hypothesis and rational expectations (Friedman, 1979; Froot, 1989; Crump et al., 2024). This is because these regressions approximate expected interest rates with their realized values, an approach that relies on the rational expectations postulate that agents' expectations are consistent with the true data-generating process. As a result, a rejection based on these tests may reflect a failure of the expectations hypothesis, the assumption of rational expectations, or both. Indeed, even the "algebraic equivalence" of different formulations of the expectations hypothesis hinges on rational expectations, which requires that investors correctly understand the relationship between short- and long-term interest rates. For instance, if market participants misperceive this relationship—or believe that others do—then the yield on a long-term bond need not equal the average of expected short-term rates, even if expected one-period holding returns are equal across maturities.[1]

In this paper, we examine how investors' subjective expectations shape the term structure of interest rates. Rather than imposing rational expectations, we allow investors to hold arbitrary (and heterogeneous) beliefs about future interest rates. We derive the relationships that expected and realized interest rates must satisfy under various assumptions on investors' expectations and develop regression-based tests for two key hypotheses: (i) that bond risk premia are constant, and (ii) that investors understand the structural relationships linking short- and long-term interest rates. Empirically, we find that short-term bonds do not exhibit time-varying risk premia, and market participants' expectations across different maturities and forecast horizons are not consistent with one another.

[1] This is because the equality of expected one-period returns is a statement about one-period-ahead forecasts, whereas the formulation of the expectations hypothesis that links long-term yields to expected future short rates involves forecasts over multiple horizons. The two formulations are equivalent only if forecasts across different horizons satisfy certain consistency conditions, as we discuss below.

We derive our results in the context of a simple discrete-time economy populated by long-lived investors who can hold positions in bonds of varying maturities. Unlike prior literature, we do not impose rational expectations; instead, we allow investors to hold arbitrary and potentially different beliefs about future bond prices. Specifically, we focus on what is often referred to as the economy's "temporary equilibrium," which imposes individual rationality and market clearing but treats expectations as model primitives (i.e., without modeling how they are formed). Accordingly, heterogeneity in investors' expectations in our framework may stem from different information sets, different models for how various (endogenous or exogenous) variables evolve, different understandings of the economy's structural equations, or any combination thereof.

Using the model, we characterize how investors' subjective beliefs shape the term structure of interest rates to a second-order approximation. We show that, as long as disagreement among investors is of the same order of magnitude as their subjective uncertainty—a condition necessary for equilibrium existence—bond prices can be expressed in terms of the short-term interest rate, terms capturing risk premia, and investors' subjective expectations of bonds' resale values next period. These expectations enter prices through a specific aggregation: bonds are priced as if the economy were populated by a representative investor holding *consensus expectations*, defined as a weighted average of individual expectations with greater weights placed on investors with higher risk-bearing capacities.

This characterization leads to an intuitive but important implication: irrespective of how expectations are formed, if bond risk premia are constant, realized and expected bond yields must satisfy

$$y_t^{(n)} = \frac{1}{n} y_t^{(1)} + \frac{n-1}{n} \overline{\mathbb{E}}_t[y_{t+1}^{(n-1)}] + \text{constant}, \qquad \text{(SL-EH)}$$

where $\overline{\mathbb{E}}_t[\cdot]$ denotes consensus expectations. We refer to this relationship, which links yields of adjacent maturities through one-period-ahead consensus forecasts, as the *subjective local expectations hypothesis* (SL-EH).[2] Crucially, this is the only relationship among interest rates and expectations that follows from individual portfolio optimization and market clearing. Any other relationship, including the standard formulation of the expectations hypothesis (EH) or its variants with subjective expectations, necessarily rests on additional assumptions on how expectations are formed.

Building on this observation, we then develop two families of regressions that use survey data to test whether bond risk premia are constant. The first is the survey-based counterpart to the textbook regression of Campbell and Shiller (1991) and tests the relationship between yield spreads and the one-period-ahead expected change in long-term yields. The second exploits the fact that, even under arbitrary expectations, one-period-ahead consensus forecasts of forward rates must equal current forward rates plus a constant risk premium. Importantly, both approaches enable us to test whether risk premia are constant without imposing any assumptions on how beliefs are formed (rational expectations or otherwise).

While the SL-EH describes the relationship between realized and one-period-ahead forecasts of yields of adjacent maturities, it is otherwise silent on how expectations across other maturities and forecast horizons relate to one another. We address this question in our second set of results, where we

[2] We borrow the term local expectations hypothesis from Cox, Ingersoll, and Ross (1981) and Campbell (1986), who consider similar relationships but under rational expectations.

examine whether expectations across the yield curve are mutually consistent—that is, whether market participants' expectations reflect the structural relationships that link different interest rates.

To this end, we characterize the relationship between realized and expected yields under the additional assumption of common knowledge of rationality: investors understand the market structure and how others form expectations, and these understandings are common knowledge.[3] We show that this assumption implies that the yield on a long-term bond equals market participants' consensus expectation of the average short rate over the bond's lifetime:

$$y_t^{(n)} = \frac{1}{n}\sum_{s=0}^{n-1} \overline{\mathbb{E}}_t[y_{t+s}^{(1)}] + \text{constant}; \qquad \text{(S-EH)}$$

a relationship we refer to the *subjective expectations hypothesis* (S-EH).

This result has two implications. First, it clarifies that while the assumption of constant risk premia—absent further restrictions—implies only the SL-EH, the algebraic equivalence between the SL-EH and S-EH rests on the additional assumption of common knowledge of rationality. Second, it enables us to develop three families of regressions that use survey data to test whether expectations across the yield curve are mutually consistent.

The first two families of regressions rely on the observation that, under the S-EH, consensus expectations of forward rates at *all* forecast horizons must equal current forward rates, up to a constant risk premium. The third family relies on the related observation that an expected increase in interest rates should translate into a contemporaneous expected rise in future forward rates. Importantly, all three regressions are tests of investors' mental models and assess whether their subjective expectations across maturities and horizons are consistent with one another, irrespective of how interest rates actually evolve. As such, they are fundamentally distinct from tests of consistency of subjective expectations with the true data-generating process (such as Coibion and Gorodnichenko (2015) and Bordalo et al. (2020)).

We then use survey data from Blue Chip Financial Forecasts (BCFF) and data on zero-coupon Treasury yields to empirically test whether expected and realized interest rates satisfy the SL-EH and S-EH. Using a panel of forecasts for yields across different maturities and forecast horizons, we obtain the following results.

First, we find no evidence of SL-EH violations at the short end of the yield curve, but clear violations at the long end: when the yield spread widens, expected long-maturity yields tend to fall rather than rise. Consistent with our theoretical framework, we interpret this pattern as evidence of time-varying risk premia for long-term bonds, while the null of constant risk premia cannot be rejected for short-term bonds. These findings align with earlier results in the literature, yet they are obtained without relying on assumptions about expectation formation (such as rational expectations).

Second, we document violations of the S-EH for bonds of all maturities, including those at the short end of the yield curve. Combined with the failure to reject SL-EH for short-maturity bonds, these findings point to violations of the common-knowledge assumption. In particular, they imply that investors' subjective expectations of interest rates across maturities and horizons are either inconsistent

[3]This assumption does not require market participants to share the same information, agree on the nature of fundamental shocks, or hold a common prior. Nor does it require their expectations to be consistent with the true data-generating process, as would be the case under rational expectations. Rather, it requires that each investor understands how others translate their information, priors, and mental models into expectations about bond prices.

with the structural properties of the economy—such as market clearing—or that these structural relationships are not common knowledge. In either case, investors' mental models are inconsistent with the S-EH: expectations of long-term bond yields do not equal the average of expected short-term rates over the bond's lifetime, even when risk premia are constant.

Related Literature. Our paper belongs to the literature that uses survey data to study the relationship between investors' subjective expectations and the term structure of interest rates. Early contributions include Friedman (1979) and Froot (1989), who argue that empirical rejections of the expectations hypothesis may reflect violations of rational expectations rather than of the hypothesis itself. More recently, Piazzesi, Salomao, and Schneider (2015) and Nagel and Xu (2025) document that subjective bond return expectations extracted from survey data are less cyclical than what is implied by predictive regressions using realized returns. We contribute to this literature on both theoretical and empirical fronts. On the theoretical side, we show how the relationship between realized and expected interest rates depends on assumptions about expectation formation, and we develop regressions to test whether bond risk premia are constant and whether investors understand the structural relationships linking different interest rates. On the empirical side, we use survey data to examine whether market participants' expectations of short- and long-term interest rates are consistent with one another.

A more recent strand of the literature examines whether the failure of the expectations hypothesis stems from misspecification in investors' beliefs about the process driving the short-term interest rate (Crump et al., 2024), or from investors' inability to learn the parameters of that process within a reasonable timeframe (Farmer et al., 2024). Like these papers, our work allows for departures from full-information rational expectations. However, we differ along two key dimensions. First, whereas those studies assume specific parametric models for how expectations are formed, our regressions are either agnostic about the expectation formation process or provide direct tests of the assumptions underlying it. Second, while the above papers focus on expectational errors about the short rate and assume that short- and long-term rates are linked through the S-EH, we test whether market participants' expectations of different interest rates are consistent with the S-EH.

Our paper also contributes to the body of works on over- and under-reaction of expectations in macroeconomics and finance (Coibion and Gorodnichenko, 2015; Bordalo et al., 2020). In the context of the bond market, and thus more directly related to our work, Cieslak (2018), d'Arienzo (2020), and Wang (2021) document systematic errors in market participants' forecasts of future interest rates. The common feature of these studies is their focus on how subjective forecasts relate to the true data-generating process. In contrast, we examine the relationship among different subjective forecasts, and in particular, the internal consistency of market participants' mental models across the yield curve.

Our paper also contributes to the theoretical literature on heterogeneous beliefs in bond markets, such as Xiong and Yan (2009), Ehling et al. (2018), and Buraschi and Whelan (2022). These papers model belief heterogeneity by assuming that investors "agree to disagree" about economic fundamentals, such as future interest rates or inflation. In addition, for analytical tractability, they restrict heterogeneity to two types (e.g., optimists and pessimists) and impose parametric assumptions on preferences and the underlying data-generating process. The tractability of our framework allows us to relax these assumptions. Using a second-order approximation, we characterize how investors' expectations, risk-bearing capacities, and subjective uncertainty jointly shape the term structure of interest rates—without imposing parametric assumptions, specifying the source of belief heterogeneity, or limiting the number of investor types.

Finally, our paper is related to recent and independent work by Shue, Townsend, and Wang (2025), who study the comovement of expected changes in short- and long-term bond yields and document that investors tend to lump different interest rates into a single, coarse category of "interest rates." Like our work, they test whether investors' expectations are consistent with the expectations hypothesis. However, their main regression specifications rely on the assumptions that interest rates have weak serial correlations and that investors hold a correctly specified model of the short rate's evolution. In contrast, our approach tests the consistency of investors' expectations with the S-EH without imposing any assumptions on yield curve dynamics or ruling out misspecification in investors' models of the short-term interest rate.

Outline. The rest of the paper is organized as follows. We present our conceptual framework in Section 2 and develop our theoretical results in Section 3. Section 4 contains our main empirical findings. The proofs and some additional results are presented in the Appendix.

2 Model

Consider a discrete-time, infinite-horizon economy, with time indexed by $t \in \{0, 1, \dots\}$. In every period, there are zero-coupon, default-free bonds with face value one that mature in $n \geq 1$ periods. Let $s_t^{(n)}$ denote the net supply of the n-period bond at time t, which is assumed to be inelastic for all $n \geq 2$. The short-term bond is supplied elastically at price $p_t^{(1)}$, which is set by the central bank. The short-term interest rate is thus given by $y_t^{(1)} = -\log p_t^{(1)}$.

The economy is populated by a unit mass of long-lived, risk-averse bond investors, indexed by $i \in [0, 1]$, who choose their bond holdings and can readjust their portfolios each period. Investor i enters the market at $t = 0$ with initial wealth $\widetilde{w}_i$. At any future point in time, the investor exits the market with probability $1 - \rho$ and is replaced by a new investor—also indexed i—who enters with the same initial wealth $\widetilde{w}_i$. As in Vayanos and Vila (2021) and Kekre, Lenel, and Mainardi (2025), we assume investors derive utility from their terminal nominal wealth at the time of exit. Unlike these papers, however, we allow investors to have arbitrary and potentially different expectations about future bond prices. These differences in expectations may reflect investors' different information sets, different models for how various (endogenous or exogenous) variables evolve, different understandings of the economy's structural equations, or some combination thereof.

The investor indexed i thus faces the following optimization problem at time t:

$$\begin{aligned} \max_{x_{i\tau}^{(n)}} \quad & (1-\rho) \sum_{\tau=t}^{\infty} \rho^{\tau-t} \mathbb{E}_{it}[u(w_{i\tau+1})] \\ \text{s.t.} \quad & w_{i\tau+1} = w_{i\tau}/p_\tau^{(1)} + \sum_{n=2}^{\infty} x_{i\tau}^{(n)} (p_{\tau+1}^{(n-1)} - p_\tau^{(n)}/p_\tau^{(1)}), \end{aligned} \tag{1}$$

where $p_\tau^{(n)}$ is the time-τ price of the bond that matures in n periods, $x_{i\tau}^{(n)}$ denotes i's position in the bond,

$w_{i\tau}$ is i's wealth at time τ, and $u(\cdot)$ denotes investors' (strictly concave and thrice-differentiable) utility function. Operator $\mathbb{E}_{it}[\cdot]$ in (1) represents investor i's subjective expectation, which—unless otherwise noted—we treat as a model primitive that need not coincide with rational expectations. In particular, $\mathbb{E}_{it}[\cdot]$ may be inconsistent with the economy's structural equations (such as market clearing) as well as with the processes driving economic fundamentals (such as shocks to the short-term interest rate or to the supply of bonds). While we do not take a stance on how expectations are formed, we assume that investors' beliefs are internally consistent, in the sense that each investor's expectations satisfy the law of iterated expectations, that is, $\mathbb{E}_{it}\mathbb{E}_{it+h}[\cdot] = \mathbb{E}_{it}[\cdot]$ for all $h \geq 0$ and all t.[4]

To analyze how investors' expectations shape the term structure of interest rates, we approximate the economy around a deterministic steady state in which (i) the short-term interest rate is constant and equal to zero in all periods, and (ii) investors face no uncertainty about future bond prices. Formally, we parameterize investors' *subjective* uncertainty by assuming

$$\mathrm{cov}_{it}(\log p_{t+h}^{(n)}, \log p_{t+s}^{(m)}) = \sigma^2 \Sigma_{t,h,s}^{(n,m)} \tag{2}$$

and approximate the economy as $\sigma \to 0$.[5] As σ approaches zero, uncertainty about future prices vanishes at a common rate σ^2. Since the right-hand side of (2) is independent of i, this specification also implies that all investors face the same subjective uncertainty at each point in time. Throughout, we assume that the subjective conditional covariance matrix of future log prices is invertible when $\sigma > 0$.

We also assume that

$$\lim_{\sigma \to 0} \frac{1}{\sigma^2}(\mathbb{E}_{it}[\log p_{t+h}^{(n)}] - \mathbb{E}_{jt}[\log p_{t+h}^{(n)}]) < \infty \tag{3}$$

for all $i \neq j$. This condition restricts the extent of disagreement between investors: while i and j may hold different beliefs, the magnitude of their disagreement shrinks to zero at rate σ^2 (or faster). Thus, in light of (2), disagreement between investors is at most of the same order of magnitude as their subjective uncertainty. When (3) is violated, investors with different beliefs would want to take infinitely sized bets against one another as $\sigma \to 0$, thereby ruling out equilibrium existence. This restriction ensures that, despite potential heterogeneity in expectations, investors' bond-holding positions remain finite as uncertainty vanishes.

We now characterize equilibrium bond prices in terms of model primitives. Let

$$\theta_{it} = -u''(w_{it})/u'(w_{it}) \qquad \text{and} \qquad \gamma_{it} = -w_{it}u''(w_{it})/u'(w_{it}) \tag{4}$$

denote investor i's time-t coefficients of absolute and relative risk aversion, respectively. Also, let $\bar{\theta}_t$ denote the harmonic average of investors' coefficients of absolute risk aversion at time t. We have the following result.

[4] The law of iterated expectations is a consistency restriction on expectations of the same variable, i.e., $\mathbb{E}_{it}\mathbb{E}_{it+h}[z] = \mathbb{E}_{it}[z]$ for any z. It places no requirement on investor i's expectations of distinct variables—say, $p_{t+s}^{(n)}$ and $p_{t+s}^{(m)}$—being consistent or even related to one another. We impose this assumption to ensure that, given an investor's expectations, the corresponding portfolio optimization problem is dynamically consistent. The assumption can be dispensed with when investors are short-lived ($\rho = 0$).

[5] The assumption that the short-term interest rate is equal to one in steady state corresponds to $\lim_{\sigma \to 0} \log p_t^{(1)} = 0$. This normalization ensures that, in steady state, each investor's wealth remains constant over time; that is, $\lim_{\sigma \to 0} w_{it} = \widetilde{w}_i$ for all t. We also assume that all (subjective) joint moments of future log prices of order $k \in \{1, 2, \dots\}$ decay to zero at rate σ^k. This guarantees that the log-quadratic approximation of equilibrium prices only depends on the first two moments of investors' beliefs.

Proposition 1. *The log price of the n-period bond at time t is given by*

$$\begin{aligned}\log p_t^{(n)} &= \log p_t^{(1)} + \overline{\mathbb{E}}_t[p_{t+1}^{(n-1)}] + \frac{1}{2}\mathrm{var}_t(\log p_{t+1}^{(n-1)}) - \bar{\theta}_t \sum_{m=2}^{\infty} s_t^{(m)} \mathrm{cov}_t(\log p_{t+1}^{(n-1)}, \log p_{t+1}^{(m-1)}) \\ &\quad - \left(\int_0^1 (\bar{\theta}_t/\theta_{it})(1-\gamma_{it})\mathrm{d}i\right) \sum_{h=1}^{\infty} \rho^h \mathrm{cov}_t(\log p_{t+1}^{(n-1)}, \log p_{t+h}^{(1)})\end{aligned} \tag{5}$$

to a second-order approximation as $\sigma \to 0$, where

$$\overline{\mathbb{E}}_t[\cdot] = \int_0^1 (\bar{\theta}_t/\theta_{it})\mathbb{E}_{it}[\cdot]\mathrm{d}i \tag{6}$$

denotes investors' consensus expectations.

This result characterizes equilibrium bond prices in terms of investors' subjective expectations, their risk-bearing capacities, the short-term interest rate, and the supply of bonds. Crucially for our purposes, Proposition 1 holds regardless of how investors form their expectations, as it follows directly from individual portfolio optimization and market clearing. In particular, the expression in (5) remains valid whether investors hold identical or heterogeneous beliefs, regardless of how their expectations relate (if at all) to the process governing the short-term interest rate, and irrespective of their understanding of the relationship among different bond prices—including equation (5) itself. As a result, this proposition allows us to study the implications of investors' subjective beliefs for the term structure of interest rates without taking an explicit or implicit stance on how those beliefs are formed.[6]

We now examine each term on the right-hand side of (5) in turn. The first two terms show that the price of the n-period bond depends on the short-term interest rate and on a weighted average of investors' expectations. Notably, despite the fact that investors are long-lived, only their one-period-ahead forecasts of the bond's resale price, $\mathbb{E}_{it}[p_{t+1}^{(n-1)}]$, enter the expression; expectations over longer horizons or those of other bond prices do not. This reflects the assumption that investors can rebalance their portfolios each period. Proposition 1 thus highlights that any other relationship among various bond prices and/or their expectations—such as the textbook statement of the expectations hypothesis relating bond prices to longer-horizon expectations of the short-term interest rate—necessarily relies on additional assumptions on how investors form expectations.

The second term on the right-hand side of (5) indicates that bonds are priced as if the economy were populated by a representative investor holding the *consensus expectations* defined in (6). These expectations are weighted averages of investors' subjective forecasts, with greater weights assigned to investors with lower coefficients of absolute risk aversion, θ_{it}. This reflects the intuitive idea that beliefs of investors with higher risk-bearing capacity exert greater influence on asset prices.[7] It is worth noting that, despite (3), the cross-sectional distribution of expectations shapes bond prices in a nontrivial way: while (3) rules out disagreements of order σ, it still allows for disagreements of order σ^2, which affect

[6] In the language of Adam and Marcet (2011), Proposition 1 relies on investors' *internal rationality*—that is, they understand their own objective and optimize their portfolios accordingly, given their subjective beliefs—but not on their *external rationality* (i.e., whether they understand the market structure, other investors' behavior, etc.).

[7] For example, in the special case of logarithmic utility, the weighted average on the right-hand side of (5) reduces to the wealth-weighted average of investors' expectations, in line with Xiong and Yan (2009). Also see Barillas and Nimark (2017), who derive an equation similar to (5) under the additional assumptions that all investors (i) are short-lived, (ii) have logarithmic preferences, and (iii) form rational expectations under incomplete information about the short-term interest rate.

consensus expectations in (6) to a second-order approximation. In other words, investor disagreement, though bounded, plays a meaningful role in determining equilibrium prices.

The third term on the right-hand side of (5) is a standard convexity adjustment that arises from expressing the relationship between different bond price in logs.

The remaining two terms on the right-hand side of (5) are also standard and capture the bond's risk premia. The fourth term represents the "myopic component" of the compensation investors demand for bearing the bond's price risk. This compensation increases with the harmonic average of investors' absolute risk aversion, the covariance between the bond's resale price, $\log p_{t+1}^{(n-1)}$, and the prices of other bonds, and investors' positions in those bonds—which, by market clearing, equal the bonds' net supply, $s_t^{(m)}$. The fifth term is a reflection of investors' intertemporal hedging demand due to uncertainty about future interest rates: conservative investors (with $\gamma_{it} > 1$) demand more of the n-period bond when its resale price tends to be high in states where future short-term interest rates are expected to be low (Campbell and Viceira, 2001). This term vanishes when all investors are either short-lived or have logarithmic preferences.

3 Expectations and the Term Structure of Interest Rates

In this section, we take equation (5) as our starting point and examine how investors' expectations shape the term structure of interest rates. Throughout, we maintain the additional assumption that the convexity adjustment term, $\frac{1}{2}\mathrm{var}_t(\log p_{t+1}^{(n-1)})$, in (5) is either small or constant over time. This ensures that fluctuations in bond prices are driven entirely by changes in risk premia, investors' expectations, or both. Unless otherwise noted, all results are presented to the same order of approximation as in Proposition 1.

3.1 Term Structure with Constant Risk Premia

As a first step, we derive the relationship between bond yields, denoted by $y_t^{(n)} = -(1/n)\log p_t^{(n)}$, and investors' expectations under the assumption that risk premia are constant. This serves as a natural benchmark akin to the textbook statement of the expectations hypothesis, but without imposing restrictions on how expectations are formed.

Assumption 1. Bond risk premia are constant over time.

Proposition 2. *If Assumption 1 is satisfied, then*

$$y_t^{(n)} = \frac{1}{n}y_t^{(1)} + \frac{n-1}{n}\overline{\mathbb{E}}_t[y_{t+1}^{(n-1)}] + \textit{constant}, \tag{7}$$

where $\overline{\mathbb{E}}_t[\cdot]$ *denotes investors' consensus expectations in* (6).

Equation (7), which follows from (5), is the counterpart to the expectations hypothesis in our heterogeneous-beliefs economy with arbitrary subjective expectations. It links yields on bonds with adjacent maturities to one another through investors' one-period-ahead consensus forecasts. Borrowing from Cox, Ingersoll, and Ross (1981) and Campbell (1986), we refer to this relationship as the *subjective local expectations hypothesis* (SL-EH).

Just as in (5), equation (7) holds regardless of how investors form their expectations, as it follows from individual portfolio optimization and market clearing. Put differently, when bond risk premia are constant, this equation remains valid irrespective of what investors may or may not know about the market structure, other market participants' behavior or expectations, and the fundamental shocks to the economy. An important consequence of this observation is that any relationship between different interest rates and/or their expectations other than (7) must hinge on additional assumptions on investors' expectations. In particular, any statement that relates $y_t^{(n)}$ to $\overline{\mathbb{E}}_t[y_{t+h}^{(m)}]$ for $h > 1$ or $m \neq n-1$ necessarily assumes either a connection between long-horizon and one-period-ahead forecasts, a specific view about how investors perceive the relationship among different yields, or both.

Equation (7) has equivalent formulations in terms of excess returns and forward rates. Specifically, it implies that, aside from a constant risk premium, the consensus expected one-period holding returns are identical across all maturities; i.e., $\overline{\mathbb{E}}_t[\mathrm{rx}_{t+1}^{(n)}]$ is a constant that does not vary with time, where $\mathrm{rx}_{t+1}^{(n)} = \log p_{t+1}^{(n-1)} - \log p_t^{(n)} - y_t^{(1)}$ denotes excess returns. Equation (7) also implies that forward rates satisfy

$$\overline{\mathbb{E}}_t[f_{t+1}^{(n,m)}] = f_t^{(n,m+1)} + \text{constant}, \tag{8}$$

where $f_t^{(n,m)} = (1 + m/n)y_t^{(n+m)} - (m/n)y_t^{(m)}$ is the current yield of an n-period bond that matures in $n + m$ periods. This relationship resembles the martingale property of forward rates (plus a constant) dating back to Samuelson (1965) and Sargent (1972). However, it differs from these classical results along two dimensions. First, it shows that, away from rational expectations, forward rates are martingales with respect to the investors' consensus *subjective* expectations, rather than the expectations implied by the true data-generating process. Second, the martingale property of forward rates holds only for one-period-ahead forecasts: in general, the difference between $\overline{\mathbb{E}}_t[f_{t+h}^{(n,m)}]$ and $f_t^{(n,m+h)}$ need not be constant for $h \geq 2$. This reflects the fact that, absent other restrictions on expectations, constant risk premia alone do not imply a relationship between current yields and investors' multi-step-ahead forecasts.

Regression specification. Equations (7) and (8) allow us to test the hypothesis that bond risk premia are constant, without requiring any assumptions on how expectations are formed.

Equation (7) implies that when risk premia are constant—and hence the SL-EH holds—the slope coefficient of regression

$$\overline{\mathbb{E}}_t[y_{t+1}^{(n-1)}] - y_t^{(n)} = \alpha + \beta_1 \frac{1}{n-1}(y_t^{(n)} - y_t^{(1)}) + \varepsilon_t \tag{9}$$

must equal 1 for all $n \geq 2$. Regression (9) closely parallels the well-known Campbell and Shiller (1991) regression, $y_{t+1}^{(n-1)} - y_t^{(n)} = \alpha + \frac{1}{n-1}\beta(y_t^{(n)} - y_t^{(1)}) + \varepsilon_t$, but with the realized yield $y_{t+1}^{(n-1)}$ on the left-hand side replaced by its time-t consensus forecast, $\overline{\mathbb{E}}_t[y_{t+1}^{(n-1)}]$. Naturally, the regression also has a similar interpretation: under the null hypothesis that risk premia are constant (i.e., $\beta_1 = 1$), an increase in the yield spread $y_t^{(n)} - y_t^{(1)}$ must be associated with a higher *expected* yield of the long-term bond to ensure that subjective expected returns of long- and short-term bonds are equalized (up to a constant). However, unlike the standard Campbell-Shiller regression, which is valid only under rational expectations, regression (9) is a valid test of Assumption 1 regardless of how expectations are formed.

Similarly, the martingale property of forward rates in equation (8) implies that under the SL-EH, the slope coefficient of regression

$$\overline{\mathbb{E}}_t[f_{t+1}^{(n,m)}] - y_t^{(n)} = \alpha + \beta_2(f_t^{(n,m+1)} - y_t^{(n)}) + \varepsilon_t \tag{10}$$

must equal 1 for all $n \geq 1$ and $m \geq 0$. This family of regressions thus provides another test of Assumption 1. In the special case that $m = 0$, regression (10) can be equivalently expressed as $\overline{\mathbb{E}}_t[\mathrm{rx}_{t+1}^{(n+1)}] = \alpha + (1 - \beta_2)(f_t^{(1,n)} - y_t^{(1)}) + \varepsilon_t$, which is the survey-based analogue of the Fama and Bliss (1987) regression, with realized excess returns replaced by their one-period-ahead consensus forecasts.

A few remarks are in order. First, while regressions (9) and (10) are equivalent under the null of constant risk premia, they constitute distinct tests of this hypothesis, as neither follows from the other without explicitly imposing Assumption 1. Second, the error terms, ε_t, in these regressions capture potential measurement error in constructing consensus expectations. The slope coefficients remain unbiased so long as this measurement error is uncorrelated with the regressors. Third, although prior studies have used survey data to test for time variation in risk premia, such tests typically rely on additional assumptions about expectation formation. For instance, Friedman (1979) tests whether $\overline{\mathbb{E}}_t[y_{t+h}^{(1)}] = f_t^{(1,h)}$, while Froot (1989) examines whether $\overline{\mathbb{E}}_t[y_{t+h}^{(n)}] - y_t^{(h)} = f_t^{(n,h)} - y_t^{(h)}$. However, except when $h = 1$, both tests rely on forecasts beyond the immediate next period, which means they are valid tests of Assumption 1 only under additional assumptions on how expectations are formed.

3.2 Consistency of Yield Curve Expectations

While the SL-EH describes the relationship between realized and one-period-ahead forecasts of yields of adjacent maturities, it is otherwise silent on how expectations across other maturities and forecast horizons relate to one another. This is because we have so far treated investors' expectations as model primitives, placing no restrictions on how forecasts are formed across the yield curve. As such, investors' subjective expectations of short- and long-term interest rates may be entirely disconnected.

In this subsection, we examine whether investors' subjective expectations are consistent with the structural equations that link different interest rates to one another. This requires imposing additional restrictions on investors' mental models—and, by extension, on how their yield curve expectations are formed.

Assumption 2. (a) Investors understand market clearing and other investors' ability to solve their own portfolio optimization problems; (b) investors understand how others form expectations; (c) these understandings are common knowledge among all investors.

Assumption 2(a) requires that all investors understand the market structure: they recognize that all market participants solve similar optimization problems—albeit with different expectations and wealth—and that bond markets must clear. As a result, each investor also understands that bond prices satisfy equation (5). Assumption 2(b) adds the requirement that investors understand how others form expectations. This does not mean that investors share the same information, agree on the nature of fundamental shocks, or hold a common prior. Nor does it require their expectations to be consistent with the true data-generating process, as would be the case under rational expectations.[8] Rather, it requires that each investor understands how others translate their information, priors, and mental models into expectations about bond prices. Finally, part (c) requires that the understanding of market structure and expectation formation is common knowledge among all market participants.

Our next result establishes the implications of Assumption 2 for the term structure of interest rates. Let $\Delta^h_{ij,t} = \mathbb{E}_{it}[\log p^{(1)}_{t+h}] - \mathbb{E}_{jt}[\log p^{(1)}_{t+h}]$ denote the disagreement between investors i and j at time t about the future price of the short-term bond h periods ahead. We have the following result.

Proposition 3. *Suppose Assumptions 1 and 2 are satisfied. Also suppose*

$$\lim_{\sigma\to 0} \frac{1}{\sigma^2}(\mathbb{E}_{it}[\Delta^h_{ij,t+1}] - \mathbb{E}_{jt}[\Delta^h_{ij,t+1}]) = 0 \tag{11}$$

for all $h \geq 1$ uniformly. Then,

$$y^{(n)}_t = \frac{1}{n}\sum_{s=0}^{n-1} \overline{\mathbb{E}}_t[y^{(1)}_{t+s}] + \textit{constant.} \tag{12}$$

Proposition 3 establishes that, under Assumptions 1 and 2, the yield on a long-term bond equals the average of investors' consensus expectations of the short rate over the bond's lifetime, plus a constant risk premium. We refer to the expression in (12) as the *subjective expectations hypothesis* (S-EH), as it mirrors the textbook statement of the expectations hypothesis but replaces expectations under the true data-generating process with consensus subjective forecasts. Comparing this result to Proposition 2 underscores that while the assumption of constant risk premia—on its own—implies only the SL-EH, the algebraic equivalence between the SL-EH and S-EH relies on the additional common knowledge assumption. Put differently, when the common knowledge assumption is violated, the SL-EH may hold even if the S-EH does not. Equation (11) is a technical condition that ensures disagreements about disagreements are sufficiently small.

The key implication of Proposition 3 is that once Assumption 2 is imposed, investors' yield curve expectations can no longer be arbitrary and must satisfy certain consistency conditions. Specifically, it is easy to verify that equation (12) implies that

$$\overline{\mathbb{E}}_t[f^{(n,m)}_{t+h}] = \overline{\mathbb{E}}_t[f^{(n,m+h-s)}_{t+s}] + \text{constant} \qquad \text{for all } h > s \geq 0. \tag{13}$$

This relationship, which generalizes the martingale property of forward rates in (8) to forecast horizons beyond the next period (i.e., $h > 1$), places tight constraints on how investors' subjective expectations across maturities and horizons relate to one another. For instance, it requires that

$$\overline{\mathbb{E}}_t[y^{(n)}_{t+1}] - y^{(n)}_t = \frac{1}{n}(\overline{\mathbb{E}}_t[y^{(1)}_{t+n}] - y^{(1)}_t) + \text{constant}, \tag{14}$$

i.e., an expected increase in the short-term interest rate must be accompanied by a contemporaneous increase in expected long-term interest rates. This is because under common knowledge (Assumption 2), investors understand that long-term rates reflect the average of expectations of the short rate. As

[8] That is, Assumption 2(b) allows for private information, heterogeneous beliefs, model misspecification, the possibility that investors "agree to disagree" about the processes driving economic fundamentals, and any combination thereof.

a result, any change in forecasts of future short rates translates into a corresponding adjustment in expected long rates.

We conclude with a brief discussion of the role of the various assumptions underpinning Proposition 3. Recall that Assumption 1 implies that bond yields satisfy equation (7). Imposing Assumption 2 in addition ensures that this relationship is common knowledge among all investors. As a result, one can iterate forward on (7) to express the yield on the n-period bond in terms of the iterated consensus expectations of future short-term interest rates:

$$y_t^{(n)} = \frac{1}{n} \sum_{s=0}^{n-1} \overline{\mathbb{E}}_t \overline{\mathbb{E}}_{t+1} \ldots \overline{\mathbb{E}}_{t+s-1} [y_{t+s}^{(1)}] + \text{constant.} \tag{15}$$

It is well understood that consensus expectations generally do not satisfy the law of iterated expectations ($\overline{\mathbb{E}}_t \overline{\mathbb{E}}_{t+h}[\cdot] \neq \overline{\mathbb{E}}_t[\cdot]$), even if individual expectations do (Allen, Morris, and Shin, 2006). Nonetheless, Proposition 3 shows that when disagreements about disagreements are sufficiently small (as formalized by (11)), the iterated consensus expectations in (15) can be approximated to a second order by first-order consensus expectations of future short rates.[9]

Regression specification. We now use equation (13) to derive three regression specifications that test whether expectations across the yield curve are consistent with the S-EH.

First, setting $s = 0$ in (13) implies that the slope coefficient of regression

$$\overline{\mathbb{E}}_t[f_{t+h}^{(n,m)}] - y_t^{(n)} = \alpha + \beta_3 (f_t^{(n,m+h)} - y_t^{(n)}) + \varepsilon_t \tag{16}$$

must equal 1 for all $n \geq 1$, $m \geq 0$, and $h \geq 2$. That is, under the null of the S-EH, an increase in the forward premium today must be accompanied by an expected increase in future forward rates. Note that we exclude one-period-ahead forecasts of forward rates: setting $h = 1$ reduces the above specification to (10), which, as discussed in Subsection 3.1, constitutes a test of the SL-EH rather than the S-EH.

Second, setting $m = 0$ in (13) implies that $\overline{\mathbb{E}}_t[y_{t+h}^{(n)}] - \overline{\mathbb{E}}_t[f_{t+s}^{(n,h-s)}]$ must be constant over time, and therefore unpredictable by any time-t variable. Accordingly, the slope coefficient of regression

$$\overline{\mathbb{E}}_t[y_{t+h}^{(n)}] - \overline{\mathbb{E}}_t[f_{t+s}^{(n,h-s)}] = \alpha + \beta_4 z_t + \varepsilon_t \tag{17}$$

must be zero for any z_t, where $h > s \geq 1$. Rejecting $\beta_4 = 0$ indicates that investors' expectations about future interest rates are inconsistent with the S-EH, even when risk premia are constant. Such a rejection may arise because investors do not understand the market structure or how others form expectations—i.e., a violation of Assumptions 2(a) and 2(b)—or because this understanding is not common knowledge among market participants, as required by Assumption 2(c). In either case, the implication is that investors' mental models are such that their expectations of $y_{t+h}^{(n)}$ and $f_{t+s}^{(n,h-s)}$ do not move in tandem as required by the S-EH.

Third and finally, it is easy to verify that the martingale property of forward rates also requires that the slope coefficient of the following regression is equal to 1:

$$\overline{\mathbb{E}}_t[f_{t+s}^{(h,n-s)}] - y_t^{(h)} = \alpha + \beta_5 \frac{n}{h} (\overline{\mathbb{E}}_t[y_{t+h}^{(n)}] - y_t^{(n)}) + \varepsilon_t \tag{18}$$

[9]Put differently, although consensus expectations generally fail to satisfy the law of iterated expectations exactly, they do so up to a second-order approximation as $\sigma \to 0$, provided condition (11) holds.

where $h \geq 1$ and $1 \leq s \leq n$. In other words, an expected rise in the yield of the n-period bond should translate into a simultaneous increase in forecasts of forward rates up to n periods into the future.

We conclude this discussion with two observations. First, note that regressions (16)–(18) test the internal consistency of investors' subjective expectations across different maturities and forecast horizons, irrespective of the actual dynamics of the yield curve. In other words, they test the compatibility of investors' mental models with the S-EH, regardless of how—or whether—their expectations relate to the true data-generating process. This stands in contrast to regressions that constitute joint tests of the expectations hypothesis and rational expectations, such as Campbell and Shiller (1991), as well as to those in the style of Coibion and Gorodnichenko (2015), which examine the relationship between subjective expectations and the actual data-generating process.

Second, in the special case that $n = s = 1$, regression (18) simplifies to

$$\overline{\mathbb{E}}_t[y_{t+s}^{(h)}] - y_t^{(h)} = \alpha + \beta_5 \frac{1}{h}(\overline{\mathbb{E}}_t[y_{t+h}^{(1)}] - y_t^{(1)}) + \varepsilon_t,$$

which tests the relationship in (14) and resembles the specification in Shue, Townsend, and Wang (2025). The difference is that while the above regression uses the expected *long-run change* in the short rate, $\overline{\mathbb{E}}_t[y_{t+h}^{(1)}] - y_t^{(1)}$, as the explanatory variable, theirs uses the expected *one-period change*, $\overline{\mathbb{E}}_t[y_{t+1}^{(1)}] - y_t^{(1)}$. This distinction is important for interpreting the slope coefficients. Under the S-EH, our specification guarantees $\beta_5 = 1$ without imposing any assumptions on yield-curve dynamics or on the relationship between investors' subjective models of the short-term interest rate and the true data-generating process. In contrast, the slope coefficient in Shue, Townsend, and Wang (2025) depends both on investors' beliefs about future path of the short-term interest rate and on the relationship between their subjective model and the true data-generating process, even under the S-EH.

3.3 Empirical Implementation of Consensus Expectations

Propositions 1–3 highlight that the term structure of interest rates is determined as if the economy were populated by a representative investor who holds the consensus beliefs defined in equation (6). These consensus expectations are weighted averages of individual forecasts, with greater weight placed on investors with higher risk-bearing capacities—that is, lower coefficients of absolute risk aversion. As a result, any empirical implementation of the regressions discussed above requires information on forecasters' coefficients of risk aversion (or, under specific assumptions on preferences, their wealth). Since such data are typically unavailable, most empirical studies instead use the unweighted average or median of forecasts (Singleton, 2021).

In anticipation of our empirical analysis in the next section, we conclude this section by providing a theoretical justification for approximating the consensus expectations in (6) by the unweighted average of investors' expectations. We say that investors are *ex ante symmetric* if they have identical wealth when they enter the market—that is, $\widetilde{w}_i = \widetilde{w}_j$ for all i and j—while still allowing for heterogeneity in expectations. We then have the following result.

Proposition 4. *Suppose all investors are ex ante symmetric. Then,*

$$\overline{\mathbb{E}}_t[y_{t+h}^{(n)}] = \int_0^1 \mathbb{E}_{it}[y_{t+h}^{(n)}]\mathrm{d}i \tag{19}$$

to a second-order approximation as $\sigma \to 0$.

This result shows that when investors are ex ante symmetric, the consensus expectations in Propositions 1–3, and the resulting regression specifications, can be approximated to a second order by the unweighted average of individual expectations. Ex ante symmetry therefore provides a theoretical justification for the common practice of using the cross-sectional average of forecasts in empirical analyses.

Notably, Proposition 4 does not require *ex post* symmetry. In fact, differences in expectations may lead investors to make different portfolio choices, which in turn may cause their wealth—and thus their risk-bearing capacities—to evolve differently over time. However, according to (19), such endogenous fluctuations in the wealth distribution do not affect bond yields to a second-order approximation. This is because equilibrium existence requires disagreements among investors to vanish at rate of σ^2 or faster (equation (3)). Consequently, starting from identical initial wealth, the cross-sectional covariance between investors' risk-bearing capacities, $1/\theta_{it}$, and their expectations, $\mathbb{E}_{it}[y_{t+h}^{(n)}]$, decays strictly faster than σ^2, implying that the wedge between the two sides of equation (19) is equal to zero to a second-order approximation as $\sigma \to 0$.

4 Empirical Analysis

In this section, we use survey and interest rate data to test the two hypotheses developed in the previous section. First, we use regressions (9) and (10) to test whether realized and expected interest rates satisfy the SL-EH. By Proposition 2, this is equivalent to testing whether bond risk premia are constant. Second, we use regressions (16)–(18) to assess whether market participants' expectations across different maturities and forecast horizons are consistent with the S-EH.

4.1 Data

Estimating regressions (9), (10), and (16)–(18) requires data on expected and realized interest rates. We therefore combine survey data with data on zero-coupon Treasury yields, as described below.

Survey data. We use forecasts from the Blue Chip Financial Forecasts (BCFF) survey. Towards the end of each month, approximately 40 financial professionals submit projections for interest rates and other macroeconomic variables. Respondents provide estimates for the current quarter and up to five quarters ahead, with forecasts released on the first day of the following month. We sample the data at a quarterly frequency from 1988:Q1 through 2024:Q4 and focus on forecasts of 1, 2, 4, 8, 20, and 40-quarter yields. Following Proposition 4, we calculate the unweighted cross-sectional average of all interest rate forecasts as a proxy for $\overline{\mathbb{E}}_t[y_{t+h}^{(n)}]$, where $h = 1, \ldots, 6$ quarters.

Interest rates. We use zero-coupon yield curve estimates from Liu and Wu (2021), who construct daily yields for maturities from 1 month to 30 years starting in 1961. We convert the series to quarterly frequency by averaging daily yields in each quarter. To align with the survey data, we restrict the sample to 1988:Q1–2024:Q4 and focus on maturities ranging from 1 to 41 quarters.

Table 1. Tests of the SL-EH Using Regression (9)

	$\overline{\mathbb{E}}_t[y_{t+1}^{(n-1)}] - y_t^{(n)}$					
	$n = 2$	$n = 3$	$n = 5$	$n = 9$	$n = 21$	$n = 41$
$\frac{1}{n-1}(y_t^{(n)} - y_t^{(1)})$	0.83	0.97	0.72	0.84	-0.22^{**}	-2.97^{***}
	(0.31)	(0.29)	(0.31)	(0.30)	(0.47)	(0.95)

Notes: This table presents the regression coefficients of specification (9). The consensus forecasts $\overline{\mathbb{E}}_t[\cdot]$ are constructed as the cross-sectional average of the forecasters in the dataset. Bond maturities are $n = 2, 3, 6, 9, 21$, and 41 quarters. The sample period runs from 1988:Q1 to 2024:Q4. The numbers in parentheses report Newey-West standard errors with lag length selected according to Lazarus, Lewis, Stock, and Watson (2018). Stars indicate statistical significance for the null hypothesis $\beta_1 = 1$, with *, **, and *** indicating statistical significance at 10%, 5%, and 1% levels, respectively.

4.2 Subjective Local Expectations Hypothesis

As our first empirical exercise, we use regressions (9) and (10) to test whether realized and expected interest rates satisfy the SL-EH, or equivalently, whether risk premia are constant. Recall that those regressions allow us to test Assumption 1 while remaining agnostic about how expectations are formed.

We begin with regression (9), the survey-based analogue of the Campbell–Shiller regression. This specification tests the relationship between yield spreads, $y_t^{(n)} - y_t^{(1)}$, and the one-period-ahead expected change in yields, $\overline{\mathbb{E}}_t[y_{t+1}^{(n-1)}] - y_t^{(n)}$. Under the null of constant risk premia, the slope coefficient of this regression must be 1, implying that a larger yield spread corresponds to a higher expected long-term yield. Because the BCFF survey only contains forecasts for bond yields of maturities $1, 2, 4, 8, 20$, and 40 quarter, we run regression (9) for $n = 2, 3, 5, 9, 21$, and 41 quarters.

Table 1 presents the results. For shorter maturities ($n = 2, 3, 5$, and 9 quarters), we find no evidence of a violation of the SL-EH—and hence no evidence for the presence of time-varying risk premia—as the null hypothesis $\beta_1 = 1$ cannot be rejected. In contrast, for longer maturities ($n = 21$ and 41 quarters), the estimated coefficients differ significantly from one and are, in fact, negative. That is, when the yield spread widens, expected long-term yields tend to decline rather than rise. This is a clear violation of the SL-EH and, according to Proposition 2, implies that long-term bonds exhibit time-varying risk premia.

Next, we turn to regression (10), which tests the one-period-ahead martingale property of forward rates implied by equation (8). Recall that under the null hypothesis of constant risk premia, the slope coefficient must equal 1 for all $n \geq 1$ and $m \geq 0$. To implement this regression, we calculate the expected forward rates from the one-period-ahead forecasts of yields: $\overline{\mathbb{E}}_t[f_{t+1}^{(n,m)}] = (1+m/n)\overline{\mathbb{E}}_t[y_{t+1}^{(n+m)}] - (m/n)\overline{\mathbb{E}}_t[y_{t+1}^{(m)}]$.

Table 2 reports the results. Estimated coefficients echo the results from Table 1. For shorter maturities (i.e., when both n and $n + m$ are small) the estimated coefficients are statistically indistinguishable from 1, indicating that an increase in forward premia is accompanied by a one-to-one rise in the difference between the (consensus) forecasts of the forward rate and the realized yield, $\overline{\mathbb{E}}_t[f_{t+1}^{(n,m)}] - y_t^{(n)}$. In contrast, for longer maturities (that is, when either m or $n + m$ are large), we can reject $\beta_2 = 1$. Another pattern that emerges is that, holding the start of the forward period (m) fixed, the estimated coefficient decreases with the bond's maturity (n).

Table 2. Martingale Property of Forward Rates with Respect to One-Period-Ahead Subjective Forecasts

time to maturity $(m+n)$ in quarters	start of the forward period (m) in quarters					
	0	1	2	4	8	20
1	0.92 (0.15)					
2	0.94 (0.14)	1.03 (0.07)				
4	0.70^{*} (0.17)	0.83^{*} (0.09)	0.86 (0.09)			
8	0.65^{**} (0.14)	0.81^{**} (0.08)	0.87^{**} (0.06)	0.95 (0.04)		
20	0.01^{***} (0.25)	0.46^{***} (0.12)	0.62^{***} (0.08)	0.76^{***} (0.05)	0.86^{***} (0.03)	
40	-2.11^{***} (0.66)	-0.60^{***} (0.32)	-0.09^{***} (0.21)	0.31^{***} (0.13)	0.58^{***} (0.09)	0.76^{***} (0.06)

Notes: This table presents the regression coefficients of specification (10). Consensus forecasts $\overline{\mathbb{E}}_t[\cdot]$ are constructed as the cross-sectional average of the forecasters in the dataset. The sample period runs from 1988:Q1 to 2024:Q4. The numbers in parentheses report Newey-West standard errors with lag length selected according to Lazarus, Lewis, Stock, and Watson (2018). Stars indicate statistical significance for the null hypothesis $\beta_2 = 1$, with *, **, and *** indicating statistical significance at 10%, 5%, and 1% levels, respectively.

Taken together, Tables 1 and 2 show that expected and realized yields are consistent with constant risk premia at the short end of the yield curve but exhibit meaningful departures from the SL-EH at the long end, indicating the presence of time-varying risk premia.

4.3 Subjective Expectations Hypothesis

As our next set of exercises, we estimate regressions (16)–(18) to assess whether investors' expectations are consistent with the S-EH. As established in Proposition 3, these specifications constitute joint tests of Assumptions 1 and 2.

Table 3 reports the estimated coefficients of regression (16) and their standard errors, pooling observations across forecast horizons $h = 2, 3, 4, 5$, and 6 quarters.[10] The estimated coefficients are all below one and, in the vast majority of cases, statistically different from unity. This pattern implies a rejection of the S-EH and, by extension, a joint rejection of Assumptions 1 and 2 across the yield curve. However, in light of the results in Tables 1 and 2—which found no evidence against Assumption 1 at the short end—we interpret the rejection of $\beta_3 = 1$ for short-maturity bonds in Table 3 primarily as evidence against Assumption 2.

Comparing the entries corresponding to the short end of the yield curve in Tables 2 and 3 illustrates this point more clearly: while Table 2 shows that forward rates are a martingale with respect to one-step-ahead subjective forecasts, Table 3 reveals that the martingale property fails for the same bonds when

[10] We exclude $h = 1$ because, at this horizon, the specification reduces to (10) and therefore tests the SL-EH rather than the S-EH.

Table 3. Martingale Property of Forward Rates

time to maturity $(m+n)$ in quarters	start of the forward period (m) in quarters: 0	1	2	4	8	20
1	0.83^{**} (0.07)					
2	0.83^{**} (0.07)	0.87^{**} (0.05)				
4	0.82^{**} (0.07)	0.84^{***} (0.05)	0.86^{***} (0.04)			
8	0.89 (0.07)	0.89^{**} (0.05)	0.91^{**} (0.04)	0.95 (0.03)		
20	0.90 (0.06)	0.88^{**} (0.05)	0.88^{**} (0.04)	0.90^{***} (0.03)	0.92^{***} (0.02)	
40	0.71^{**} (0.11)	0.68^{***} (0.09)	0.68^{***} (0.07)	0.71^{***} (0.06)	0.76^{***} (0.04)	0.83^{***} (0.02)

Notes: This table presents the regression coefficients of specification (16), pooled across forecast horizons $h = 2, 3, 4, 5$, and 6 quarters. Consensus forecasts $\overline{\mathbb{E}}_t[\cdot]$ are constructed as the cross-sectional average of the forecasters in the dataset. The sample period runs from 1988:Q1 to 2024:Q4. The numbers in parentheses report Newey-West standard errors with lag length selected according to Lazarus, Lewis, Stock, and Watson (2018). Stars indicate statistical significance for the null hypothesis $\beta_3 = 1$, with *, **, and *** indicating statistical significance at 10%, 5%, and 1% levels, respectively.

forecast horizons extend beyond one period. This discrepancy indicates that subjective expectations across maturities and horizons are either inconsistent with the structural properties of the economy, such as market clearing (violating Assumptions 2(a) and (b)), or that these structural relationships are not common knowledge (violating Assumption 2(c)).

We next turn to regression (17), which tests the S-EH by examining whether the difference $\overline{\mathbb{E}}_t[y^{(n)}_{t+h}] - \overline{\mathbb{E}}_t[f^{(n,h-s)}_{t+s}]$ is predictable using a time-t variable z_t. Because our survey data cover only a limited set of maturities and forecast horizons, we can implement this regression only in the case $s = h-n$. As in Fama and Bliss (1987), we use the forward premium as the predictor, resulting in the following specification:

$$\overline{\mathbb{E}}_t[y^{(n)}_{t+h}] - \overline{\mathbb{E}}_t[f^{(n,n)}_{t+h-n}] = \alpha + \beta_4(f^{(n,1)}_t - y^{(n)}_t) + \varepsilon_t. \tag{20}$$

Table 4 reports the estimated slope coefficients for individual forecast horizons h, as well as estimates from regressions pooled across all horizons for each maturity n. The estimates reject the null hypothesis $\beta_4 = 0$, indicating that investors' expectations of $y^{(n)}_{t+h}$ and $f^{(n,n)}_{t+h-n}$ do not move in tandem in response to changes in forward premia. In line with the results in Table 3, these findings provide further evidence that investors' expectations are inconsistent with the S-EH.

Finally, we turn to regression (18) as another test of whether investors understand the relationship between short- and long-term interest rates. This regression tests whether expected changes in the short-rate translate to higher expected changes in long-term interest rates. The results are presented in Table 5. Since we only have forecasts up to five quarters ahead, we can only run this regression for bonds of maturity $n = 2$ and 4 quarters. In both cases, we find a coefficient that is greater than 1, though

Table 4. Tests of the S-EH Using Regression (20)

maturity (n) in quarters	forecast horizon (h) in quarters					
	2	3	4	5	6	pooled
1	2.87^{*} (1.46)	3.26^{**} (1.39)	3.42^{**} (1.32)	3.45^{**} (1.26)	2.47^{*} (1.27)	3.18^{***} (0.47)
2		-0.39^{*} (0.20)	-0.59^{**} (0.23)	-0.68^{**} (0.26)	-0.31 (0.21)	-0.52^{***} (0.10)
4				-11.93^{**} (4.15)	-4.58 (3.67)	-9.71^{**} (3.40)

Notes: This table presents the regression coefficients of specification (20). The last column reports the estimates for regressions pooled across all forecast horizons $h = 2, \ldots, 6$ quarters. Consensus forecasts $\overline{\mathbb{E}}_t[\cdot]$ are constructed as the cross-sectional average of the forecasters in the dataset. The sample period runs from 1988:Q1 to 2024:Q4. The numbers in parentheses report Newey-West standard errors with lag length selected according to Lazarus, Lewis, Stock, and Watson (2018). Stars indicate statistical significance for the null hypothesis $\beta_4 = 0$, with *, **, and *** indicating statistical significance at 10%, 5%, and 1% levels, respectively.

only the estimate for $n = 2$ quarters is statistically different from 1.

5 Conclusion

A large body of empirical research has documented strong evidence against the expectations hypothesis of interest rates. This rejection is typically interpreted as evidence of time-varying risk premia. However, standard tests of the expectations hypothesis are inherently joint tests of that hypothesis and of rational expectations.

In this paper, we examine how investors' subjective expectations shape the term structure of interest rates without imposing rational expectations. We allow investors to hold arbitrary subjective beliefs and derive the no-arbitrage relationships that realized and expected rates must satisfy under constant risk premia. We then show how survey data can be used both to construct regression-based tests of constant risk premia and to assess whether investors understand the linkages between short- and long-term interest rates.

Empirically, we find that (i) short-term bonds exhibit no time-varying risk premia, yet (ii) investors' long-term interest rate forecasts remain inconsistent with the expectations hypothesis—even under their own subjective expectations of the short rate. These results suggest that robust rejections of the expectations hypothesis partly reflect violations of rational expectations rather than genuine time-varying risk premia.

Table 5. Testing for Relationship Between Interest Rates using Regression (18)

	$\overline{\mathbb{E}}_t[y^{(n)}_{t+1}] - y^{(n)}_t$	
	$n = 2$	$n = 4$
$\frac{1}{n}(\overline{\mathbb{E}}_t[y^{(1)}_{t+n}] - y^{(1)}_t)$	1.25***	1.21
	(0.05)	(0.13)

Notes: This figure reports OLS regression coefficients from equation (18) regressing (consensus) expected change in yields, $\overline{\mathbb{E}}_t[y^{(n)}_{t+1}] - y^{(n)}_t$, on (consensus) expected change in short-term yields, $\frac{1}{n}(\overline{\mathbb{E}}_t[y^{(1)}_{t+n}] - y^{(1)}_t)$. Consensus forecasts are constructed as the average of the forecasters in the dataset. Bond maturities are $n = 2, 4$, quarters. The sample period runs from 1988:Q1 to 2024:Q4. The numbers in parentheses report Newey-West standard errors with lag length selected according to Lazarus, Lewis, Stock, and Watson (2018). Stars indicate statistical significance for the null hypothesis that $\beta = 1$. *, **, and *** indicate statistical significance at 10%, 5%, and 1% levels, respectively.

A Appendix

Proof of Proposition 1

Consider the optimization problem (1) of investor i at time t with wealth w_{it}. Denoting the Lagrange multiplier corresponding to the investor's time-τ budget constraint by $(1-\rho)\rho^{t-\tau}\lambda_{i\tau+1}$, the first-order optimality conditions are given by

$$\lambda_{i\tau+1} = u'_i(w_{i\tau+1}) + \rho\mathbb{E}_{i\tau+1}[\lambda_{i\tau+2}]/p^{(1)}_{\tau+1} \tag{A.1}$$

and

$$\mathbb{E}_{i\tau}[\lambda_{i\tau+1}(p^{(n-1)}_{\tau+1} - p^{(n)}_\tau/p^{(1)}_\tau)] = 0 \tag{A.2}$$

for all $\tau \geq t$. Solving for $\lambda_{i\tau+1}$ from (A.1) recursively and using the transversality condition, we have

$$\lambda_{i\tau+1} = \sum_{h=0}^{\infty} \rho^h \mathbb{E}_{i\tau+1}\left[\frac{u'_i(w_{i\tau+h+1})}{p^{(1)}_{\tau+1}\cdots p^{(1)}_{\tau+h}}\right]. \tag{A.3}$$

Furthermore, multiplying the budget constraint in (1) by $\lambda_{i\tau+1}$ and using (A.2) implies that

$$\mathbb{E}_{i\tau}[\lambda_{i\tau+1}w_{i\tau+1}] = \mathbb{E}_{i\tau}[\lambda_{i\tau+1}]w_{i\tau}/p^{(1)}_\tau. \tag{A.4}$$

We use the above equations to derive a second-order approximation to log bond prices as $\sigma \to 0$. But first, we characterize the steady-state values of w_{it+h} and $\mathbb{E}_{it}[\lambda_{it+1}]$. Recall from (2) that when $\sigma = 0$, investor i faces no subjective uncertainty regarding the future and believe that the short-term interest rate is held constant and equal to one in all future periods $\tau \geq t+1$. Thus, (A.4) implies that $\lim_{\sigma\to 0} w_{it+h+1} = w_{it}$ for all $h \geq 0$. This, together with (A.3) then implies that

$$\lim_{\sigma\to 0}\mathbb{E}_{it}[\lambda_{it+1}] = \frac{u'_i(w_{it})}{1-\rho}. \tag{A.5}$$

With the above in hand, we next obtain a second-order approximation to (A.2) as $\sigma \to 0$. Note that setting $\tau = t$ in (A.2) implies that

$$\text{cov}_{it}(\lambda_{it+1}, p^{(n-1)}_{t+1}) + \mathbb{E}_{it}[\lambda_{it+1}](\mathbb{E}_{it}[p^{(n-1)}_{t+1}] - p^{(n)}_t/p^{(1)}_t) = 0. \tag{A.6}$$

Equation (A.3), combined with the assumption that investors' expectations satisfy the law of iterated expectations, leads to

$$\text{cov}_{it}(\lambda_{it+1}, p_{t+1}^{(n-1)}) = \sum_{h=0}^{\infty} \rho^h \text{cov}_{it}\left(\frac{u_i'(w_{it+h+1})}{p_{t+1}^{(1)} \cdots p_{t+h}^{(1)}}, p_{t+1}^{(n-1)}\right),$$

and as a result,

$$\begin{aligned}\text{cov}_{it}(\lambda_{it+1}, p_{t+1}^{(n-1)}) &= u_i''(w_{it}) \sum_{h=0}^{\infty} \rho^h \text{cov}_{it}(w_{it+h+1}, \log p_{t+1}^{(n-1)}) \\ &\quad - u_i'(w_{it}) \sum_{h=1}^{\infty} \rho^h \sum_{r=1}^{h} \text{cov}_{it}(\log p_{t+r}^{(1)}, \log p_{t+1}^{(n-1)}) + o(\sigma^2)\end{aligned} \tag{A.7}$$

to a second-order approximation as $\sigma \to 0$, where we are using the fact that when $\sigma = 0$, investor i believes that $\lim_{\sigma\to 0} w_{it+h+1} = w_{it}$. Next, note that setting $\tau = t + h + 1$ in (A.4) implies that

$$w_{it+h+1} = \frac{\mathbb{E}_{it+h+1}[\lambda_{it+h+2} w_{it+h+2}] p_{t+h+1}^{(1)}}{\mathbb{E}_{it+h+1}[\lambda_{it+h+2}]}$$

for all $h \geq 0$. Consequently,

$$\text{cov}_{it}(w_{it+h+1}, \log p_{t+1}^{(n-1)}) = \text{cov}_{it}(w_{it+h+2}, \log p_{t+1}^{(n-1)}) + w_{it}\text{cov}_{it}(\log p_{t+h+1}^{(1)}, \log p_{t+1}^{(n-1)}) + o(\sigma^2).$$

Using this recursion to solve for $\text{cov}_{it}(w_{it+h+1}, \log p_{t+1}^{(n-1)})$ in terms of $\text{cov}_{it}(w_{it+1}, \log p_{t+1}^{(n-1)})$, we obtain

$$\text{cov}_{it}(w_{it+h+1}, \log p_{t+1}^{(n-1)}) = \text{cov}_{it}(w_{it+1}, \log p_{t+1}^{(n-1)}) - w_{it} \sum_{r=0}^{h-1} \text{cov}_{it}(\log p_{t+r+1}^{(1)}, \log p_{t+1}^{(n-1)}) + o(\sigma^2)$$

for all $h \geq 1$. Plugging the above into the first term on the right-hand side of (A.7) implies that

$$\begin{aligned}\text{cov}_{it}(\lambda_{it+1}, p_{t+1}^{(n-1)}) = -\frac{u_i'(w_{it})}{1-\rho}\Bigg(&\theta_{it}\text{cov}_{it}(w_{it+1}, \log p_{t+1}^{(n-1)}) \\ &+ (1-\gamma_{it}) \sum_{h=1}^{\infty} \rho^h \text{cov}_{it}(\log p_{t+h}^{(1)}, \log p_{t+1}^{(n-1)})\Bigg) + o(\sigma^2),\end{aligned}$$

where θ_{it} and γ_{it} denote, respectively, investor i's coefficients of absolute and relative risk aversion at time t, defined in (4). Combining the above equation with (A.6) and replacing for w_{it+1} from the budget constraint in (1), we obtain

$$\begin{aligned}\mathbb{E}_{it}[p_{t+1}^{(n-1)}] - p_t^{(n)}/p_t^{(1)} &= \theta_{it} \sum_{m=2}^{\infty} x_{it}^{(m)} \text{cov}_t(\log p_{t+1}^{(m-1)}, \log p_{t+1}^{(n-1)}) \\ &\quad + (1-\gamma_{it}) \sum_{h=1}^{\infty} \rho^h \text{cov}_t(\log p_{t+h}^{(1)}, \log p_{t+1}^{(n-1)}) + o(\sigma^2),\end{aligned} \tag{A.8}$$

where we are also using the assumption that $\text{cov}_{it}(\log p_{t+h}^{(n)}, \log p_{t+h}^{(m)})$ is independent of i (equation (2)), and the fact that $\lim_{\sigma\to 0} \mathbb{E}_{it}[\lambda_{it+1}] = u_i'(w_{it})/(1-\rho)$ (equation (A.5)). Dividing both sides of the above equation by θ_{it}, integrating over all i, and imposing market clearing, we obtain

$$p_t^{(n)}/p_t^{(1)} = \int_0^1 (\theta_{it}/\bar{\theta}_t)^{-1}\mathbb{E}_{it}[p_{t+1}^{(n-1)}]\mathrm{d}i - \bar{\theta}_t \sum_{m=2}^{\infty} s_t^{(m)}\mathrm{cov}_t(\log p_{t+1}^{(m-1)}, \log p_{t+1}^{(n-1)}) - \left(\int_0^1 (\theta_{it}/\bar{\theta}_t)^{-1}(1-\gamma_{it})\mathrm{d}i\right)\sum_{h=1}^{\infty}\rho^h \mathrm{cov}_t(\log p_{t+h}^{(1)}, \log p_{t+1}^{(n-1)}) + o(\sigma^2),$$

where $\bar{\theta}_t$ is the harmonic average of investors' coefficients of absolute risk aversion at time t. Taking logarithms from both sides of the above equation leads to

$$\log p_t^{(n)} = \log p_t^{(1)} + \int_0^1 (\theta_{it}/\bar{\theta}_t)^{-1}\mathbb{E}_{it}[\log p_{t+1}^{(n-1)}]\mathrm{d}i + \frac{1}{2}\mathrm{var}_t(\log p_{t+1}^{(n-1)}) - \bar{\theta}_t \sum_{m=2}^{\infty} s_t^{(m)}\mathrm{cov}_t(\log p_{t+1}^{(m-1)}, \log p_{t+1}^{(n-1)}) - \left(\int_0^1 (\theta_{it}/\bar{\theta}_t)^{-1}(1-\gamma_{it})\mathrm{d}i\right)\sum_{h=1}^{\infty}\rho^h \mathrm{cov}_t(\log p_{t+h}^{(1)}, \log p_{t+1}^{(n-1)}) + \frac{1}{2}\int_0^1 (\theta_{it}/\bar{\theta}_t)^{-1}\mathbb{E}_{it}^2[\log p_{t+1}^{(n-1)}]\mathrm{d}i - \frac{1}{2}\left(\int_0^1 (\theta_{it}/\bar{\theta}_t)^{-1}\mathbb{E}_{it}[\log p_{t+1}^{(n-1)}]\mathrm{d}i\right)^2 + o(\sigma^2).$$

The last two terms on the right-hand side of the above equation reflect the cross-sectional covariance of investors' forecasts of next-period prices, with higher weights assigned to investors with smaller coefficients of relative risk aversion. However, recall from (3) that the disagreement between investors shrinks to zero at a rate faster than σ. As a result, this cross-sectional variance term decays to zero at rate faster than σ^2, thus establishing (5).

To complete the proof, we verify that equilibrium quantities $x_{it}^{(m)}$ remain bounded for all investors i as $\sigma \to 0$. To this end, note that equation (A.8) implies that

$$\mathbb{E}_{it}[p_{t+1}^{(n-1)}] - \mathbb{E}_{jt}[p_{t+1}^{(n-1)}] = \sum_{m=2}^{\infty}(\theta_{it}x_{it}^{(m)} - \theta_{jt}x_{jt}^{(m)})\mathrm{cov}_t(\log p_{t+1}^{(m-1)}, \log p_{t+1}^{(n-1)}) + (\gamma_{jt} - \gamma_{it})\sum_{h=1}^{\infty}\rho^h \mathrm{cov}_t(\log p_{t+h}^{(1)}, \log p_{t+1}^{(n-1)}) + o(\sigma^2)$$

for all $i \neq j$. Recall from (2) that $\mathrm{cov}_t(\log p_{t+h}^{(n)}, \log p_{t+s}^{(m)}) = \sigma^2\Sigma_{t,h,s}^{(n,m)}$. Therefore, if we let $\mathbf{\Sigma}_t$ denote the matrix with (n,m) element given by $\Sigma_{t,1,1}^{(n,m)}$, the above equation implies that

$$\theta_{it}x_{it}^{(m)} - \theta_{jt}x_{jt}^{(m)} = \frac{1}{\sigma^2}\sum_{n=1}^{\infty}(\mathbf{\Sigma}_t^{-1})_{m-1,n}(\mathbb{E}_{it}[p_{t+1}^{(n)}] - \mathbb{E}_{jt}[p_{t+1}^{(n)}]) - (\gamma_{jt} - \gamma_{it})\frac{1}{\sigma^2}\sum_{h=1}^{\infty}\rho^h\sum_{n=1}^{\infty}(\mathbf{\Sigma}_t^{-1})_{m-1,n}\mathrm{cov}_t(\log p_{t+h}^{(1)}, \log p_{t+1}^{(n)}) + o(\sigma^2)$$

for all $m \geq 2$. Note that assumptions (2) and (3) guarantee that the right-hand side of above the equation remains finite as $\sigma \to 0$. As a result, $\theta_{it}x_{it}^{(m)} - \theta_{jt}x_{jt}^{(m)} = O(1)$ for all i and j. Dividing both sides by θ_{jt}, integrating over all j, and using market clearing then guarantees that $\lim_{\sigma\to 0}|x_{it}^{(m)}| < \infty$ for all $m \geq 2$. □

Proof of Proposition 2

Under Assumption 1, the last two terms on the right-hand side (5) do not vary with time. This, together with our maintained assumption that the convexity adjustment term is also constant (or small), implies that $\log p_t^{(n)} = \log p_t^{(1)} + \overline{\mathbb{E}}_t[p_{t+1}^{(n-1)}] + \text{constant}$. Expressing this equation in terms of bond yields establishes (7). $\square$

Proof of Proposition 3

By Proposition 2, bond yields satisfy equation (7) to a second-order approximation as $\sigma \to 0$. Since each investor i understands market clearing and all other investors' ability to solve their optimization problems (Assumption 2(a)), it follows that $\mathbb{E}_{it}[y_{t+1}^{(n-1)}] = \frac{1}{n-1}\mathbb{E}_{it}[y_{t+1}^{(1)}] + \frac{n-2}{n-1}\mathbb{E}_{it}\overline{\mathbb{E}}_{t+1}[y_{t+2}^{(n-2)}] + \text{constant}$. Multiplying both sides by $\bar{\theta}_t/\theta_{it}$ and integrating over all i, it follows that

$$\overline{\mathbb{E}}_t[y_{t+1}^{(n-1)}] = \frac{1}{n-1}\overline{\mathbb{E}}_t[y_{t+1}^{(1)}] + \frac{n-2}{n-1}\overline{\mathbb{E}}_t\overline{\mathbb{E}}_{t+1}[y_{t+2}^{(n-2)}] + \text{constant}.$$

Substituting the above into (7) implies that

$$y_t^{(n)} = \frac{1}{n}y_t^{(1)} + \frac{1}{n}\overline{\mathbb{E}}_t[y_{t+1}^{(1)}] + \frac{n-2}{n}\overline{\mathbb{E}}_t\overline{\mathbb{E}}_{t+1}[y_{t+2}^{(n-2)}] + \text{constant}.$$

Since market clearing and investors' individual rationality are common knowledge (Assumption 2(c)), we can repeat the above steps iteratively, allowing us to express the yield on the n-period bond as

$$y_t^{(n)} = \frac{1}{n}\sum_{s=0}^{n-1}\overline{\mathbb{E}}_t\overline{\mathbb{E}}_{t+1}\ldots\overline{\mathbb{E}}_{t+s-1}[y_{t+s}^{(1)}] + \text{constant}$$

to a second-order approximation as $\sigma \to 0$. The proof is complete once we show that

$$\overline{\mathbb{E}}_t\overline{\mathbb{E}}_{t+1}\ldots\overline{\mathbb{E}}_{t+s-1}[y_{t+s}^{(1)}] = \overline{\mathbb{E}}_t[y_{t+s}^{(1)}] + o(\sigma^2) \tag{A.9}$$

as $\sigma \to 0$ for all $s \geq 1$.

To establish (A.9), first observe that

$$\overline{\mathbb{E}}_t\overline{\mathbb{E}}_{t+1}\ldots\overline{\mathbb{E}}_{t+s-1}[y_{t+s}^{(1)}] - \overline{\mathbb{E}}_t[y_{t+s}^{(1)}] = \sum_{k=0}^{s-2}\overline{\mathbb{E}}_t\ldots\overline{\mathbb{E}}_{t+k-1}\Big[\overline{\mathbb{E}}_{t+k}\overline{\mathbb{E}}_{t+k+1}[y_{t+s}^{(1)}] - \overline{\mathbb{E}}_{t+k}[y_{t+s}^{(1)}]\Big]. \tag{A.10}$$

Next, note that, by the definition of consensus expectations in (6), it follows that

$$\overline{\mathbb{E}}_{t+k}\overline{\mathbb{E}}_{t+k+1}[y_{t+s}^{(1)}] - \overline{\mathbb{E}}_{t+k}[y_{t+s}^{(1)}] = \int_0^1\int_0^1 \omega_{it+k}\mathbb{E}_{it+k}\Big[\omega_{jt+k+1}\Big(\mathbb{E}_{jt+k+1}[y_{t+s}^{(1)}] - \mathbb{E}_{it+k+1}[y_{t+s}^{(1)}]\Big)\Big]\mathrm{d}j\mathrm{d}i,$$

where $\omega_{it} = \bar{\theta}_t/\theta_{it}$. By a simple change of variables,

$$\begin{aligned}\overline{\mathbb{E}}_{t+k}\overline{\mathbb{E}}_{t+k+1}[y_{t+s}^{(1)}] - \overline{\mathbb{E}}_{t+k}[y_{t+s}^{(1)}] = &\int_0^1\int_i^1 \omega_{it+k}\mathbb{E}_{it+k}\Big[\omega_{jt+k+1}\Big(\mathbb{E}_{jt+k+1}[y_{t+s}^{(1)}] - \mathbb{E}_{it+k+1}[y_{t+s}^{(1)}]\Big)\Big]\mathrm{d}j\mathrm{d}i \\ &- \int_0^1\int_i^1 \omega_{jt+k}\mathbb{E}_{jt+k}\Big[\omega_{it+k+1}\Big(\mathbb{E}_{jt+k+1}[y_{t+s}^{(1)}] - \mathbb{E}_{it+k+1}[y_{t+s}^{(1)}]\Big)\Big]\mathrm{d}j\mathrm{d}i.\end{aligned}$$

As a result,

$$\overline{\mathbb{E}}_{t+k}\overline{\mathbb{E}}_{t+k+1}[y_{t+s}^{(1)}] - \overline{\mathbb{E}}_{t+k}[y_{t+s}^{(1)}] = \int_{j>i}(\omega_{it+k}\mathbb{E}_{it+k}[\omega_{jt+k+1}\Delta_{ij,t+k+1}^{s-k-1}] - \omega_{jt+k}\mathbb{E}_{jt+k}[\omega_{it+k+1}\Delta_{ij,t+k+1}^{s-k-1}])\mathrm{d}j\mathrm{d}i,$$

where $\Delta^h_{ij,t} = \mathbb{E}_{it}[\log p^{(1)}_{t+h}] - \mathbb{E}_{jt}[\log p^{(1)}_{t+h}]$ denotes the disagreement between investors i and j at time t about the future price of the short-term bond h periods ahead. Replacing the above into (A.10), dividing both sides by σ^2, and taking the limit as $\sigma \to 0$, we obtain

$$\lim_{\sigma\to 0}\frac{1}{\sigma^2}\Big(\overline{\mathbb{E}}_t\ldots\overline{\mathbb{E}}_{t+s-1}[y^{(1)}_{t+s}] - \overline{\mathbb{E}}_t[y^{(1)}_{t+s}]\Big) = \lim_{\sigma\to 0}\frac{1}{\sigma^2}\sum_{k=0}^{s-2}\overline{\mathbb{E}}_t\ldots\overline{\mathbb{E}}_{t+k-1}\int_{j>i}\Big(\omega_{it+k}\mathbb{E}_{it+k}[\omega_{jt+k+1}\Delta^{s-k-1}_{ij,t+k+1}] - \omega_{jt+k}\mathbb{E}_{jt+k}[\omega_{it+k+1}\Delta^{s-k-1}_{ij,t+k+1}]\Big)\mathrm{d}j\mathrm{d}i.$$

Recall from (2) that when $\sigma = 0$, investor i faces no uncertainty regarding the future and believes that the short-term interest rate is held constant and equal to one in all future periods $\tau \geq t+1$ (i.e., $\lim_{\sigma\to 0} p^{(1)}_t = 1$). Therefore, (A.4) implies that $\lim_{\sigma\to 0} w_{it} = \widetilde{w}_i$ for all t. This, together with the fact that $\Delta^{s-k-1}_{ij,t+k+1} = O(\sigma^2)$ as $\sigma \to 0$ implies that

$$\lim_{\sigma\to 0}\frac{1}{\sigma^2}\Big(\overline{\mathbb{E}}_t\ldots\overline{\mathbb{E}}_{t+s-1}[y^{(1)}_{t+s}] - \overline{\mathbb{E}}_t[y^{(1)}_{t+s}]\Big) = \lim_{\sigma\to 0}\frac{1}{\sigma^2}\sum_{k=0}^{s-2}\overline{\mathbb{E}}_t\ldots\overline{\mathbb{E}}_{t+k-1}\int_{j>i}\widetilde{\omega}_i\widetilde{\omega}_j\widehat{\Delta}^{s-k-1}_{ij,t+k}\mathrm{d}j\mathrm{d}i,$$

where $\widehat{\Delta}^h_{ij,t} = \mathbb{E}_{it}[\Delta^h_{ij,t+1}] - \mathbb{E}_{jt}[\Delta^h_{ij,t+1}]$ denotes the time-t disagreement between investors i and j regarding their disagreement next period. By (11), the right-hand side of the above equality is equal to zero, thus establishing (A.9). □

Proof of Proposition 4

Recall from (2) that when $\sigma = 0$, investor i faces no uncertainty regarding the future and believes that the short-term interest rate is held constant and equal to one in all future periods $\tau \geq t+1$ (i.e., $\lim_{\sigma\to 0} p^{(1)}_t = 1$). Therefore, (A.4) implies that $\lim_{\sigma\to 0} w_{it} = \widetilde{w}_i$ for all t. Since, by assumption all investors are ex ante symmetric (i.e., $\widetilde{w}_i = \widetilde{w}_j$), this implies that, in steady-state, all investors have identical risk-bearing capacities at all time periods; that is, $\lim_{\sigma\to 0}(1/\theta_{it} - 1/\bar{\theta}_t) = 0$ for all t, or equivalently,

$$1/\theta_{it} - 1/\bar{\theta}_t = o(1), \tag{A.11}$$

as $\sigma \to 0$. Next, observe that

$$\int_0^1 (1/\theta_{it})\mathbb{E}_{it}[y^{(n)}_{t+h}]\mathrm{d}i = \int_0^1 (1/\theta_{it} - 1/\bar{\theta}_t)\Big(\mathbb{E}_{it}[y^{(n)}_{t+h}] - \int_0^1 \mathbb{E}_{jt}[y^{(n)}_{t+h}]\mathrm{d}j\Big)\mathrm{d}i + (1/\bar{\theta}_t)\int_0^1 \mathbb{E}_{jt}[y^{(n)}_{t+h}]\mathrm{d}j.$$

The first term on the right-hand side of the above equation is the cross-sectional covariance of investors' risk bearing capacities, $1/\theta_{it}$, and subjective expectations, $\mathbb{E}_{it}[y^{(n)}_{t+h}]$. The second term is the product of the cross-sectional average of investors' expectations and the cross-sectional average of their risk-bearing capacities, where recall $\bar{\theta}_t$ denotes the harmonic mean of investors' coefficients of absolute risk aversion. Consequently,

$$\int_0^1 (\bar{\theta}_t/\theta_{it})\mathbb{E}_{it}[y^{(n)}_{t+h}]\mathrm{d}i - \int_0^1 \mathbb{E}_{it}[y^{(n)}_{t+h}]\mathrm{d}i = \bar{\theta}_t\int_0^1 (1/\theta_{it} - 1/\bar{\theta}_t)\Big(\mathbb{E}_{it}[y^{(n)}_{t+h}] - \int_0^1 \mathbb{E}_{jt}[y^{(n)}_{t+h}]\mathrm{d}j\Big)\mathrm{d}i.$$

Equation (3) implies that $\mathbb{E}_{it}[y^{(n)}_{t+h}] - \int_0^1 \mathbb{E}_{jt}[y^{(n)}_{t+h}]\mathrm{d}j = O(\sigma^2)$ as $\sigma \to 0$. This, together with (A.11), therefore implies that the right-hand side of the above equation decays to zero at a rate strictly faster than σ^2 as

$\sigma \to 0$. In other words,

$$\int_0^1 (\bar{\theta}_t/\theta_{it})\mathbb{E}_{it}[y_{t+h}^{(n)}]\mathrm{d}i - \int_0^1 \mathbb{E}_{it}[y_{t+h}^{(n)}]\mathrm{d}i = o(\sigma^2)$$

as $\sigma \to 0$, thus establishing (19). □

References

Adam, Klaus and Albert Marcet (2011), "Internal rationality, imperfect market knowledge and asset prices." *Journal of Economic Theory*, 146(3), 1224–1252.

Allen, Franklin, Stephen Morris, and Hyun Song Shin (2006), "Beauty contests and iterated expectations in asset markets." *Review of Financial Studies*, 19(3), 719–752.

Barillas, Francisco and Kristoffer P. Nimark (2017), "Speculation and the term structure of interest rates." *Review of Financial Studies*, 30(11), 4003–4037.

Bordalo, Pedro, Nicola Gennaioli, Yueran Ma, and Andrei Shleifer (2020), "Overreaction in macroeconomic expectations." *American Economic Review*, 110(9), 2748–2782.

Buraschi, Andrea and Paul Whelan (2022), "Speculation, sentiment, and interest rates." *Management Science*, 68(3), 2308–2329.

Campbell, John Y. (1986), "A defense of traditional hypotheses about the term structure of interest rates." *The Journal of Finance*, 41(1), 183–193.

Campbell, John Y. and Robert J. Shiller (1991), "Yield spreads and interest rate movements: A bird's eye view." *Review of Economic Studies*, 58(3), 495–514.

Campbell, John Y. and Luis M. Viceira (2001), "Who should buy long-term bonds?" *American Economic Review*, 91(1), 99–127.

Cieslak, Anna (2018), "Short-rate expectations and unexpected returns in treasury bonds." *The Review of Financial Studies*, 31(9), 3265–3306.

Coibion, Olivier and Yuriy Gorodnichenko (2015), "Information rigidity and the expectations formation process: A simple framework and new facts." *American Economic Review*, 105(8), 2644–2678.

Cox, John C., Jonathan E. Ingersoll, and Stephen A. Ross (1981), "A re-examination of traditional hypotheses about the term structure of interest rates." *The Journal of Finance*, 36(4), 769–799.

Crump, Richard K., Stefano Eusepi, and Emanuel Moench (2024), "Is there hope for the expectations hypothesis?" Federal Reserve Bank of New York Staff Reports, No. 1098.

d'Arienzo, Daniele (2020), "Maturity increasing overreaction and bond market puzzles." Working paper.

Ehling, Paul, Michael Gallmeyer, Christian Heyerdahl-Larsen, and Philipp Illeditsch (2018), "Disagreement about inflation and the yield curve." *Journal of Financial Economics*, 127(3), 459–484.

Fama, Eugene F. and Robert R. Bliss (1987), "The information in long-maturity forward rates." *American Economic Review*, 77(4), 680–692.

Farmer, Leland, Emi Nakamura, and Jon Steinsson (2024), "Learning about the long run." *Journal of Political Economy*, 132(10), 86–120.

Friedman, Benjamin M. (1979), "Interest rate expectations versus forward rates: Evidence from and expectations survey." *The Journal of Finance*, 34(4), 965–973.

Froot, Kenneth A. (1989), "New hope for the expectations hypothesis of the term structure of interest rates." *Journal of Finance*, 42(2), 283–305.

Kekre, Rohan, Moritz Lenel, and Federico Mainardi (2025), "Monetary policy, segmentation, and the term structure." NBER Working Paper No. 32324.

Lazarus, Eben, Daniel J. Lewis, James H. Stock, and Mark W. Watson (2018), "HAR inference: Recommendations for practice." *Journal of Business and Economic Statistics*, 36, 541–559.

Liu, Yan and Cynthia Jing Wu (2021), "Reconstructing the Yield Curve." *Journal of Financial Economics*, 142(3), 1395–1425.

Nagel, Stefan and Zhengyang Xu (2025), "Dynamics of subjective risk premia." *Journal of Financial Economics*, 150(2), 103713.

Piazzesi, Monika, Juliana Salomao, and Martin Schneider (2015), "Trend and cycle in bond premia." Working Paper, Stanford University.

Samuelson, Paul A. (1965), "Proof that properly anticipated prices fluctuate randomly." *Industrial Management Review*, 6, 41–49.

Sargent, Thomas J. (1972), "Rational expectations and the term structure of interest rates." *Journal of Money, Credit and Banking*, 4(1), 74–97.

Shue, Kelly, Richard Townsend, and Chen Wang (2025), "Categorical thinking about interest rates." Working paper.

Singleton, Kenneth J. (2021), "Presidential address: How much "rationality" is there in bond-market risk premiums?" *The Journal of Finance*, 76(4), 1611–1654.

Vayanos, Dimitri and Jean-Luc Vila (2021), "A preferred-habitat model of the term structure of interest rates." *Econometrica*, 89(1), 77–112.

Wang, Chen (2021), "Under- and overreaction in yield curve expectations." Working paper.

Xiong, Wei and Hongjun Yan (2009), "Heterogeneous expectations and bond markets." *The Review of Financial Studies*, 23(4), 1433–1466.